

# 10 Circles

## What You'll Learn

- **Lessons 10-1** Identify parts of a circle and solve problems involving circumference.
- **Lessons 10-2, 10-3, 10-4, and 10-6** Find arc and angle measures in a circle.
- **Lessons 10-5 and 10-7** Find measures of segments in a circle.
- **Lesson 10-8** Write the equation of a circle.

## Key Vocabulary

- chord (p. 522)
- circumference (p. 523)
- arc (p. 530)
- tangent (p. 552)
- secant (p. 561)

## Why It's Important

A circle is a unique geometric shape in which the angles, arcs, and segments intersecting that circle have special relationships. You can use a circle to describe a safety zone for fireworks, a location on Earth seen from space, and even a rainbow. *You will learn about angles of a circle when satellites send signals to Earth in Lesson 10-6.*



# Getting Started

**Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 10.

## For Lesson 10-1

## Solve Equations

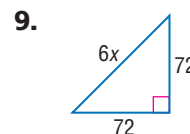
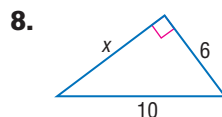
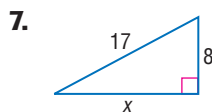
Solve each equation for the given variable. (For review, see pages 737 and 738.)

- $\frac{4}{9}p = 72$  for  $p$
- $6.3p = 15.75$
- $3x + 12 = 8x$  for  $x$
- $7(x + 2) = 3(x - 6)$
- $C = 2pr$  for  $r$
- $r = \frac{C}{6.28}$  for  $C$

## For Lesson 10-5

## Pythagorean Theorem

Find  $x$ . Round to the nearest tenth if necessary. (For review, see Lesson 7-2.)



## For Lesson 10-7

## Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth.

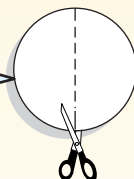
- $x^2 - 4x = 10$
- $3x^2 - 2x - 4 = 0$
- $x^2 = x + 15$
- $2x^2 + x = 15$

## FOLDABLES™ Study Organizer

**Circles** Make this Foldable to help you organize information about circles. Begin with five sheets of plain  $8\frac{1}{2}$ " by 11" paper, and cut out five large circles that are the same size.

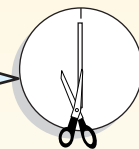
### Step 1 Fold and Cut

Fold two of the circles in half and cut one-inch slits at each end of the folds.



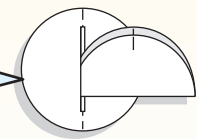
### Step 2 Fold and Cut

Fold the remaining three circles in half and cut a slit in the middle of the fold.



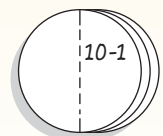
### Step 3 Slide

Slide the two circles with slits on the ends through the large slit of the other circles.



### Step 4 Label

Label the cover with the title of the chapter and each sheet with a lesson number.



**Reading and Writing** As you read and study each lesson, take notes, and record concepts on the appropriate page of your Foldable.



**What** You'll Learn

- Identify and use parts of circles.
- Solve problems involving the circumference of a circle.

**Vocabulary**

- circle
- center
- chord
- radius
- diameter
- circumference
- pi ( $\pi$ )

**How** far does a carousel animal travel in one rotation?

The largest carousel in the world still in operation is located in Spring Green, Wisconsin. It weighs 35 tons and contains 260 animals, none of which is a horse! The rim of the carousel base is a circle. The width, or diameter, of the circle is 80 feet. The distance that one of the animals on the outer edge travels can be determined by special segments in a circle.

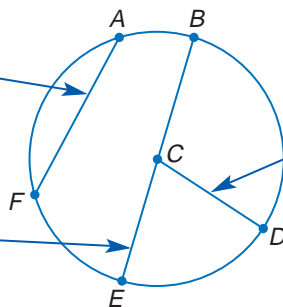
**Study Tip****Reading Mathematics**

The plural of radius is *radii*, pronounced RAY-dee-eye. The term *radius* can mean a segment or the measure of that segment. This is also true of the term *diameter*.

**PARTS OF CIRCLES** A **circle** is the locus of all points in a plane equidistant from a given point called the **center** of the circle. A circle is usually named by its center point. The figure below shows circle  $C$ , which can be written as  $\odot C$ . Several special segments in circle  $C$  are also shown.

Any segment with endpoints that are on the circle is a **chord** of the circle.  $\overline{AF}$  and  $\overline{BE}$  are chords.

A chord that passes through the center is a **diameter** of the circle.  $\overline{BE}$  is a diameter.



Any segment with endpoints that are the center and a point on the circle is a **radius**.  $\overline{CD}$ ,  $\overline{CB}$ , and  $\overline{CE}$  are radii of the circle.

Note that diameter  $\overline{BE}$  is made up of collinear radii  $\overline{CB}$  and  $\overline{CE}$ .

**Example 1** Identify Parts of a Circle

- a. Name the circle.

The circle has its center at  $K$ , so it is named circle  $K$ , or  $\odot K$ .

In this textbook, the center of a circle will always be shown in the figure with a dot.

- b. Name a radius of the circle.

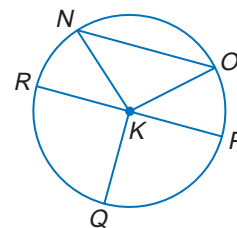
Five radii are shown:  $\overline{KN}$ ,  $\overline{KO}$ ,  $\overline{KP}$ ,  $\overline{KQ}$ , and  $\overline{KR}$ .

- c. Name a chord of the circle.

Two chords are shown:  $\overline{NO}$  and  $\overline{RP}$ .

- d. Name a diameter of the circle.

$\overline{RP}$  is the only chord that goes through the center, so  $\overline{RP}$  is a diameter.



## Study Tip

### Radii and Diameters

There are an infinite number of radii in each circle. Likewise, there are an infinite number of diameters.

By the definition of a circle, the distance from the center to any point on the circle is always the same. Therefore, all radii are congruent. A diameter is composed of two radii, so all diameters are congruent. The letters  $d$  and  $r$  are usually used to represent diameter and radius in formulas. So,  $d = 2r$  and  $r = \frac{d}{2}$  or  $\frac{1}{2}d$ .

### Example 2 Find Radius and Diameter

Circle  $A$  has diameters  $\overline{DF}$  and  $\overline{PG}$ .

- a. If  $DF = 10$ , find  $DA$ .

$$r = \frac{1}{2}d \quad \text{Formula for radius}$$

$$r = \frac{1}{2}(10) \text{ or } 5 \quad \text{Substitute and simplify.}$$

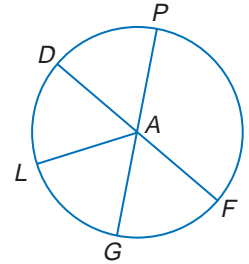
- b. If  $PA = 7$ , find  $PG$ .

$$d = 2r \quad \text{Formula for diameter}$$

$$d = 2(7) \text{ or } 14 \quad \text{Substitute and simplify.}$$

- c. If  $AG = 12$ , find  $LA$ .

Since all radii are congruent,  $LA = AG$ . So,  $LA = 12$ .



Circles can intersect. The segment connecting the centers of the two intersecting circles contains the radii of the two circles.

## Study Tip

### Congruent Circles

The circles shown in Example 3 have different radii. They are *not* congruent circles. For two circles to be *congruent circles*, they must have congruent radii or congruent diameters.

### Example 3 Find Measures in Intersecting Circles

The diameters of  $\odot A$ ,  $\odot B$ , and  $\odot C$  are 10 inches, 20 inches, and 14 inches, respectively.

- a. Find  $XB$ .

Since the diameter of  $\odot A$  is 10,  $AX = 5$ .

Since the diameter of  $\odot B$  is 20,  $AB = 10$  and  $BC = 10$ .

$\overline{XB}$  is part of radius  $\overline{AB}$ .

$$AX + XB = AB \quad \text{Segment Addition Postulate}$$

$$5 + XB = 10 \quad \text{Substitution}$$

$$XB = 5 \quad \text{Subtract 5 from each side.}$$

- b. Find  $BY$ .

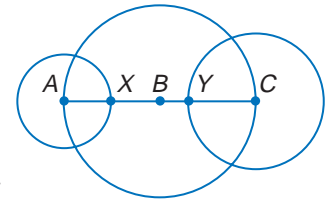
$\overline{BY}$  is part of  $\overline{BC}$ .

Since the diameter of  $\odot C$  is 14,  $YC = 7$ .

$$BY + YC = BC \quad \text{Segment Addition Postulate}$$

$$BY + 7 = 10 \quad \text{Substitution}$$

$$BY = 3 \quad \text{Subtract 7 from each side.}$$



**CIRCUMFERENCE** The **circumference** of a circle is the distance around a circle. Circumference is most often represented by the letter  $C$ .





## Geometry Activity

### Circumference Ratio

A special relationship exists between the circumference of a circle and its diameter.

#### Gather Data and Analyze

Collect ten round objects.

1. Measure the circumference and diameter of each object using a millimeter measuring tape. Record the measures in a table like the one at the right.
2. Compute the value of  $\frac{C}{d}$  to the nearest hundredth for each object. Record the result in the fourth column of the table.

Object	C	d	$\frac{C}{d}$
1			
2			
3			
:			
10			

#### Make a Conjecture

3. What seems to be the relationship between the circumference and the diameter of the circle?



### Study Tip

#### Value of $\pi$

In this book, we will use a calculator to evaluate expressions involving  $\pi$ . If no calculator is available, 3.14 is a good estimate for  $\pi$ .

The Geometry Activity suggests that the circumference of any circle can be found by multiplying the diameter by a number slightly larger than 3. By definition, the ratio  $\frac{C}{d}$  is an irrational number called **pi**, symbolized by the Greek letter  $\pi$ . Two formulas for the circumference can be derived using this definition.

$$\frac{C}{d} = \pi \quad \text{Definition of pi}$$

$$C = \pi d \quad \text{Multiply each side by } d.$$

$$C = \pi d$$

$$C = \pi(2r) \quad d = 2r$$

$$C = 2\pi r \quad \text{Simplify.}$$

### Key Concept

### Circumference

For a circumference of  $C$  units and a diameter of  $d$  units or a radius of  $r$  units,  
 $C = \pi d$  or  $C = 2\pi r$ .

If you know the diameter or radius, you can find the circumference. Likewise, if you know the circumference, you can find the diameter or radius.

### Example 4 Find Circumference, Diameter, and Radius

- a. Find  $C$  if  $r = 7$  centimeters.

$$C = 2\pi r \quad \text{Circumference formula}$$

$$= 2\pi(7) \quad \text{Substitution}$$

$$= 14\pi \text{ or about } 43.98 \text{ cm}$$

- b. Find  $C$  if  $d = 12.5$  inches.

$$C = \pi d \quad \text{Circumference formula}$$

$$= \pi(12.5) \quad \text{Substitution}$$

$$= 12.5\pi \text{ or } 39.27 \text{ in.}$$

- c. Find  $d$  and  $r$  to the nearest hundredth if  $C = 136.9$  meters.

$$C = \pi d \quad \text{Circumference formula}$$

$$136.9 = \pi d \quad \text{Substitution}$$

$$\frac{136.9}{\pi} = d \quad \text{Divide each side by } \pi.$$

$$43.58 \approx d \quad \text{Use a calculator.}$$

$$d \approx 43.58 \text{ m}$$

$$r = \frac{1}{2}d \quad \text{Radius formula}$$

$$\approx \frac{1}{2}(43.58) \quad d \approx 43.58$$

$$\approx 21.79 \text{ m} \quad \text{Use a calculator.}$$

You can also use other geometric figures to help you find the circumference of a circle.

**Standardized Test Practice**

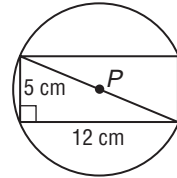
A B C D

**Example 5 Use Other Figures to Find Circumference**

**Multiple-Choice Test Item**

Find the exact circumference of  $\odot P$ .

- (A) 13 cm
- (B)  $12\pi$  cm
- (C) 40.84 cm
- (D)  $13\pi$  cm



**Read the Test Item**

You are given a figure that involves a right triangle and a circle. You are asked to find the exact circumference of the circle.

**Solve the Test Item**

The diameter of the circle is the same as the hypotenuse of the right triangle.

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

$$5^2 + 12^2 = c^2 \quad \text{Substitution}$$

$$169 = c^2 \quad \text{Simplify.}$$

$$13 = c \quad \text{Take the square root of each side.}$$

So the diameter of the circle is 13 centimeters.

$$C = \pi d \quad \text{Circumference formula}$$

$$C = \pi(13) \text{ or } 13\pi \quad \text{Substitution}$$

Because we want the exact circumference, the answer is D.



**Test-Taking Tip**

Notice that the problem asks for an exact answer. Since you know that an exact circumference contains  $\pi$ , you can eliminate choices A and C.

**Check for Understanding**

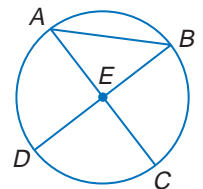
**Concept Check**

- Describe how the value of  $\pi$  can be calculated.
- Write two equations that show how the diameter of a circle is related to the radius of a circle.
- OPEN ENDED** Explain why a diameter is the longest chord of a circle.

**Guided Practice**

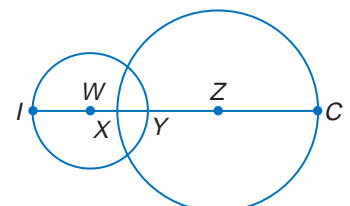
For Exercises 4–9, refer to the circle at the right.

- Name the circle.
- Name a radius.
- Name a chord.
- Name a diameter.
- Suppose  $BD = 12$  millimeters. Find the radius of the circle.
- Suppose  $CE = 5.2$  inches. Find the diameter of the circle.



Circle W has a radius of 4 units,  $\odot Z$  has a radius of 7 units, and  $XY = 2$ . Find each measure.

- $YZ$
- $IX$
- $IC$

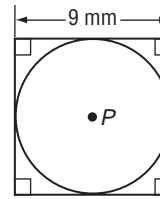


The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.

13.  $r = 5$  m,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$       14.  $C = 2368$  ft,  $d = \underline{\quad ? \quad}$ ,  $r = \underline{\quad ? \quad}$

**Standardized Test Practice**

15. Find the exact circumference of the circle.
- (A)  $4.5\pi$  mm  
 (B)  $9\pi$  mm  
 (C)  $18\pi$  mm  
 (D)  $81\pi$  mm



**Practice and Apply**

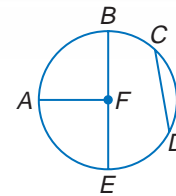
**Homework Help**

For Exercises	See Examples
16–25	1
26–31	2
32–43	3
48–51	4
52	5

**Extra Practice**  
See page 773.

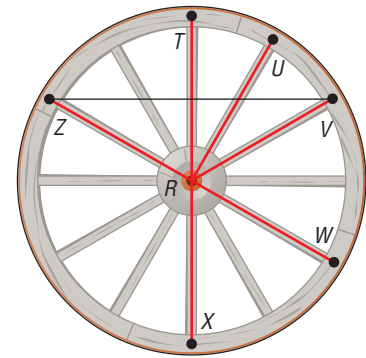
For Exercises 16–20, refer to the circle at the right.

16. Name the circle.  
 17. Name a radius.  
 18. Name a chord.  
 19. Name a diameter.  
 20. Name a radius not contained in a diameter.



**HISTORY** For Exercises 21–31, refer to the model of the Conestoga wagon wheel.

21. Name the circle.  
 22. Name a radius of the circle.  
 23. Name a chord of the circle.  
 24. Name a diameter of the circle.  
 25. Name a radius not contained in a diameter.  
 26. Suppose the radius of the circle is 2 feet. Find the diameter.  
 27. The larger wheel of the wagon was often 5 or more feet tall. What is the radius of a 5-foot wheel?  
 28. If  $TX = 120$  centimeters, find  $TR$ .  
 29. If  $RZ = 32$  inches, find  $ZW$ .  
 30. If  $UR = 18$  inches, find  $RV$ .  
 31. If  $XT = 1.2$  meters, find  $UR$ .



**More About**



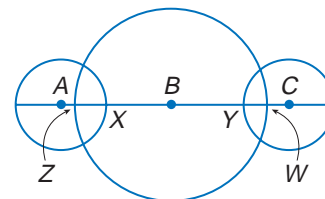
**History**

In the early 1800s, thousands of people traveled west in Conestoga wagons. The spoke wheels enabled the wagons to carry cargo over rugged terrain. The number of spokes varied with the size of the wheel.

Source: Smithsonian Institute

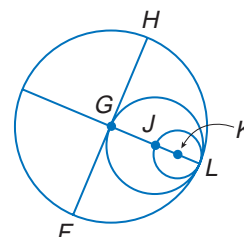
The diameters of  $\odot A$ ,  $\odot B$ , and  $\odot C$  are 10, 30, and 10 units, respectively. Find each measure if  $AZ \cong CW$  and  $CW = 2$ .

32.  $AZ$                       33.  $ZX$   
 34.  $BX$                       35.  $BY$   
 36.  $YW$                       37.  $AC$



Circles  $G$ ,  $J$ , and  $K$  all intersect at  $L$ . If  $GH = 10$ , find each measure.

38.  $FG$                       39.  $FH$   
 40.  $GL$                       41.  $GJ$   
 42.  $JL$                       43.  $JK$

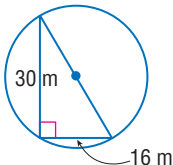
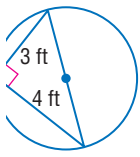
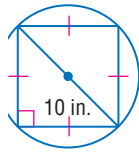
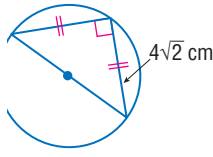




The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.

44.  $r = 7$  mm,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$       45.  $d = 26.8$  cm,  $r = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$   
 46.  $C = 26\pi$  mi,  $d = \underline{\quad ? \quad}$ ,  $r = \underline{\quad ? \quad}$       47.  $C = 76.4$  m,  $d = \underline{\quad ? \quad}$ ,  $r = \underline{\quad ? \quad}$   
 48.  $d = 12\frac{1}{2}$  yd,  $r = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$       49.  $r = 6\frac{3}{4}$  in.,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$   
 50.  $d = 2a$ ,  $r = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$       51.  $r = \frac{a}{6}$ ,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$

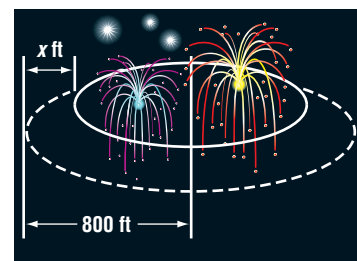
Find the exact circumference of each circle.

52.       53.       54.       55. 


56. **PROBABILITY** Find the probability that a segment with endpoints that are the center of the circle and a point on the circle is a radius. Explain.  
 57. **PROBABILITY** Find the probability that a chord that does not contain the center of a circle is the longest chord of the circle.

**FIREWORKS** For Exercises 58–60, use the following information.

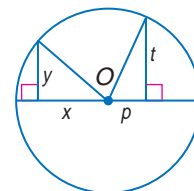
Every July 4th Boston puts on a gala with the Boston Pops Orchestra, followed by a huge fireworks display. The fireworks are shot from a barge in the river. There is an explosion circle inside which all of the fireworks will explode. Spectators sit outside a safety circle that is 800 feet from the center of the fireworks display.



58. Find the approximate circumference of the safety circle.  
 59. If the safety circle is 200 to 300 feet farther from the center than the explosion circle, find the range of values for the radius of the explosion circle.  
 60. Find the least and maximum circumference of the explosion circle to the nearest foot.

 **Online Research Data Update** Find the largest firework ever made. How does its dimension compare to the Boston display? Visit [www.geometryonline.com/data\\_update](http://www.geometryonline.com/data_update) to learn more.

61. **CRITICAL THINKING** In the figure,  $O$  is the center of the circle, and  $x^2 + y^2 + p^2 + t^2 = 288$ . What is the exact circumference of  $\odot O$ ?



62. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How far does a carousel animal travel in one rotation?**

Include the following in your answer:

- a description of how the circumference of a circle relates to the distance traveled by the animal, and
- whether an animal located one foot from the outside edge of the carousel travels a mile when it makes 22 rotations for each ride.

### Web Quest

Drawing a radius and circle on the map is the last clue to help you find the hidden treasure. Visit [www.geometryonline.com/webquest](http://www.geometryonline.com/webquest) to continue work on your WebQuest project.

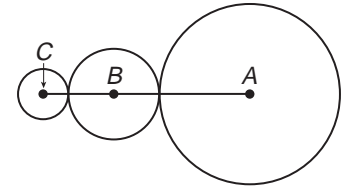




**Standardized  
Test Practice**



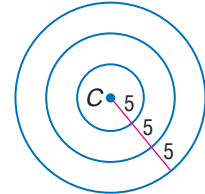
63. **GRID IN** In the figure, the radius of circle  $A$  is twice the radius of circle  $B$  and four times the radius of circle  $C$ . If the sum of the circumferences of the three circles is  $42\pi$ , find the measure of  $\overline{AC}$ .



64. **ALGEBRA** There are  $k$  gallons of gasoline available to fill a tank. After  $d$  gallons have been pumped, what percent of gasoline, in terms of  $k$  and  $d$ , has been pumped?
- (A)  $\frac{100d}{k}\%$       (B)  $\frac{k}{100d}\%$       (C)  $\frac{100k}{d}\%$       (D)  $\frac{100k-d}{k}\%$

**Extending  
the Lesson**

65. **CONCENTRIC CIRCLES** Circles that have the same center, but different radii, are called *concentric circles*. Use the figure at the right to find the exact circumference of each circle. List the circumferences in order from least to greatest.



**Maintain Your Skills**

**Mixed Review**

Find the magnitude to the nearest tenth and direction to the nearest degree of each vector. (Lesson 9-6)

66.  $\overline{AB} = \langle 1, 4 \rangle$       67.  $\vec{v} = \langle 4, 9 \rangle$   
 68.  $\overline{AB}$  if  $A(4, 2)$  and  $B(7, 22)$       69.  $\overline{CD}$  if  $C(0, -20)$  and  $D(40, 0)$

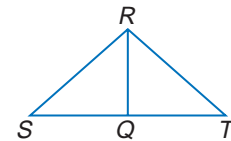
Find the measure of the dilation image of  $\overline{AB}$  for each scale factor  $k$ . (Lesson 9-5)

70.  $AB = 5, k = 6$       71.  $AB = 16, k = 1.5$       72.  $AB = \frac{2}{3}, k = -\frac{1}{2}$

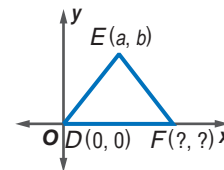
73. **PROOF** Write a two-column proof. (Lesson 5-3)

**Given:**  $\overline{RQ}$  bisects  $\angle SRT$ .

**Prove:**  $m\angle SQR > m\angle SRQ$



74. **COORDINATE GEOMETRY** Name the missing coordinates if  $\triangle DEF$  is isosceles with vertex angle  $E$ . (Lesson 4-3)



**Getting Ready for  
the Next Lesson**

**PREREQUISITE SKILL** Find  $x$ . (To review **angle addition**, see Lesson 1-4.)

75.      76.      77.
78.      79.      80.

# 10-2

# Angles and Arcs

## Vocabulary

- central angle
- arc
- minor arc
- major arc
- semicircle

### What You'll Learn

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

### What kinds of angles do the hands on a clock form?

Most clocks on electronic devices are digital, showing the time as numerals. Analog clocks are often used in decorative furnishings and wrist watches. An analog clock has moving hands that indicate the hour, minute, and sometimes the second. This clock face is a circle. The three hands form three central angles of the circle.

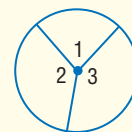


**ANGLES AND ARCS** In Chapter 1, you learned that a degree is  $\frac{1}{360}$  of the circular rotation about a point. This means that the sum of the measures of the angles about the center of the clock above is 360. Each of the angles formed by the clock hands is called a central angle. A **central angle** has the center of the circle as its vertex, and its sides contain two radii of the circle.

### Key Concept

### Sum of Central Angles

- **Words** The sum of the measures of the central angles of a circle with no interior points in common is 360.
- **Example**  $m\angle 1 + m\angle 2 + m\angle 3 = 360$



### Example 1 Measures of Central Angles

**ALGEBRA** Refer to  $\odot O$ .

a. Find  $m\angle AOD$ .

$\angle AOD$  and  $\angle DOB$  are a linear pair, and the angles of a linear pair are supplementary.

$$m\angle AOD + m\angle DOB = 180$$

$$m\angle AOD + m\angle DOC + m\angle COB = 180 \quad \text{Angle Sum Theorem}$$

$$25x + 3x + 2x = 180 \quad \text{Substitution}$$

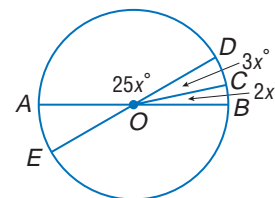
$$30x = 180 \quad \text{Simplify.}$$

$$x = 6 \quad \text{Divide each side by 60.}$$

Use the value of  $x$  to find  $m\angle AOD$ .

$$m\angle AOD = 25x \quad \text{Given}$$

$$= 25(6) \text{ or } 150 \quad \text{Substitution}$$



b. Find  $m\angle AOE$ .

$\angle AOE$  and  $\angle AOD$  form a linear pair.

$$m\angle AOE + m\angle AOD = 180 \quad \text{Linear pairs are supplementary.}$$

$$m\angle AOE + 150 = 180 \quad \text{Substitution}$$

$$m\angle AOE = 30 \quad \text{Subtract 150 from each side.}$$

A central angle separates the circle into two parts, each of which is an **arc**. The measure of each arc is related to the measure of its central angle.

### Study Tip

#### Naming Arcs

Do not assume that because an arc is named by three letters that it is a semicircle or major arc. You can also correctly name a minor arc using three letters.

### Key Concept

### Arcs of a Circle

Type of Arc:	minor arc	major arc	semicircle
Example:			
Named:	usually by the letters of the two endpoints $\widehat{AC}$	by the letters of the two endpoints and another point on the arc $\widehat{DFE}$	by the letters of the two endpoints and another point on the arc $\widehat{JML}$ and $\widehat{JKL}$
Arc Degree Measure Equals:	the measure of the central angle and is less than 180 $m\angle ABC = 110$ , so $m\widehat{AC} = 110$	360 minus the measure of the minor arc and is greater than 180 $m\widehat{DFE} = 360 - m\widehat{DE}$ $m\widehat{DFE} = 360 - 60$ or 300	$360 \div 2$ or 180 $m\widehat{JML} = 180$ $m\widehat{JKL} = 180$

Arcs in the same circle or in congruent circles with same measure are congruent.

### Theorem 10.1

In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

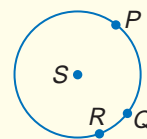
You will prove Theorem 10.1 in Exercise 54.

Arcs of a circle that have exactly one point in common are *adjacent arcs*. Like adjacent angles, the measures of adjacent arcs can be added.

### Postulate 10.1

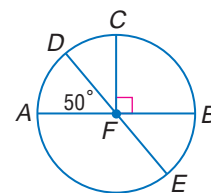
**Arc Addition Postulate** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example: In  $\odot S$ ,  $m\widehat{PQ} + m\widehat{QR} = m\widehat{PQR}$ .



### Example 2 Measures of Arcs

In  $\odot F$ ,  $m\angle DFA = 50$  and  $\overline{CF} \perp \overline{FB}$ . Find each measure.



a.  $m\widehat{BE}$

$\widehat{BE}$  is a minor arc, so  $m\widehat{BE} = m\angle BFE$ .

$\angle BFE \cong \angle DFA$  Vertical angles are congruent.

$m\angle BFE = m\angle DFA$  Definition of congruent angles

$m\widehat{BE} = m\angle DFA$  Transitive Property

$m\widehat{BE} = 50$  Substitution

b.  $m\widehat{CBE}$

$\widehat{CBE}$  is composed of adjacent arcs,  $\widehat{CB}$  and  $\widehat{BE}$ .

$m\widehat{CB} = m\angle CFB$

$= 90$

$\angle CFB$  is a right angle.

$m\widehat{CBE} = m\widehat{CB} + m\widehat{BE}$  Arc Addition Postulate

$m\widehat{CBE} = 90 + 50$  or  $140$  Substitution

c.  $m\widehat{ACE}$

One way to find  $m\widehat{ACE}$  is by using  $\widehat{ACB}$  and  $\widehat{BE}$ .

$\widehat{ACB}$  is a semicircle.

$m\widehat{ACE} = m\widehat{ACB} + m\widehat{BE}$  Arc Addition Postulate

$m\widehat{ACE} = 180 + 50$  or  $230$  Substitution

In a circle graph, the central angles divide a circle into wedges to represent data, often expressed as a percent. The size of the angle is proportional to the percent.

### Example 3 Circle Graphs

**FOOD** Refer to the graphic.

a. Find the measurement of the central angle for each category.

The sum of the percents is 100% and represents the whole.

Use the percents to determine what part of the whole circle ( $360^\circ$ ) each central angle contains.

$$2\%(360^\circ) = 7.2^\circ$$

$$6\%(360^\circ) = 21.6^\circ$$

$$28\%(360^\circ) = 100.8^\circ$$

$$43\%(360^\circ) = 154.8^\circ$$

$$15\%(360^\circ) = 54^\circ$$

$$4\%(360^\circ) = 14.4^\circ$$

b. Use the categories to identify any arcs that are congruent.

The arcs for the wedges named *Once a month* and *Less than once a month* are congruent because they both represent 2% or  $7.2^\circ$  of the circle.



**Log on for:**

- Updated data
- More activities on circle graphs

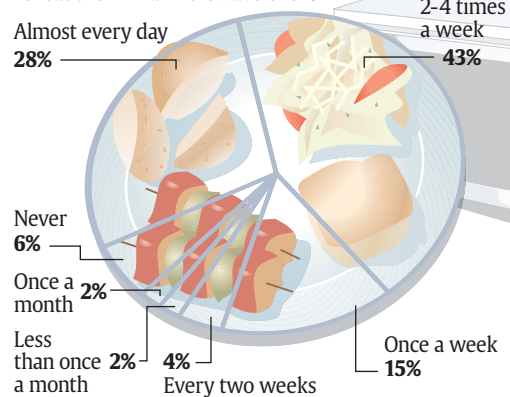
[www.geometryonline.com/usa\\_today](http://www.geometryonline.com/usa_today)



USA TODAY Snapshots®

#### Majority microwave leftovers

How often adults say they save leftovers and reheat them in a microwave oven:



Source: Opinion Research Corporation International for Tupperware Corporation

By Cindy Hall and Keith Simmons, USA TODAY





**ARC LENGTH** Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

### Study Tip

#### Look Back

To review proportions, see Lesson 6-1.

### Example 4 Arc Length

In  $\odot P$ ,  $PR = 15$  and  $m\angle QPR = 120$ . Find the length of  $\widehat{QR}$ .

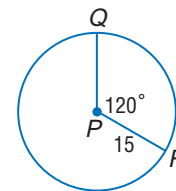
In  $\odot P$ ,  $r = 15$ , so  $C = 2\pi(15)$  or  $30\pi$  and  $m\widehat{QR} = m\angle QPR$  or 120. Write a proportion to compare each part to its whole.

$$\begin{array}{l} \text{degree measure of arc} \rightarrow \frac{120}{360} = \frac{\ell}{30\pi} \leftarrow \text{arc length} \\ \text{degree measure of whole circle} \rightarrow \frac{120}{360} = \frac{\ell}{30\pi} \leftarrow \text{circumference} \end{array}$$

Now solve the proportion for  $\ell$ .

$$\begin{array}{l} \frac{120}{360} = \frac{\ell}{30\pi} \\ \frac{120}{360}(30\pi) = \ell \quad \text{Multiply each side by } 30\pi. \\ 10\pi = \ell \quad \text{Simplify.} \end{array}$$

The length of  $\widehat{QR}$  is  $10\pi$  units or about 31.42 units.



The proportion used to find the arc length in Example 4 can be adapted to find the arc length in any circle.

### Key Concept

### Arc Length

$$\begin{array}{l} \text{degree measure of arc} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \text{arc length} \\ \text{degree measure of whole circle} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \text{circumference} \end{array}$$

$$\text{This can also be expressed as } \frac{A}{360} \cdot C = \ell.$$

## Check for Understanding

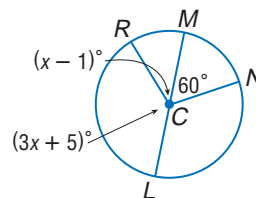
### Concept Check

- OPEN-ENDED** Draw a circle and locate three points on the circle. Name all of the arcs determined by the three points and use a protractor to find the measure of each arc.
- Explain** why it is necessary to use three letters to name a semicircle.
- Describe** the difference between *concentric* circles and *congruent* circles.

### Guided Practice

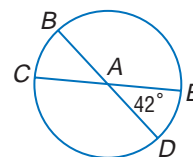
**ALGEBRA** Find each measure.

- $m\angle NCL$
- $m\angle RCM$
- $m\angle RCL$
- $m\angle RCN$



In  $\odot A$ ,  $m\angle EAD = 42$ . Find each measure.

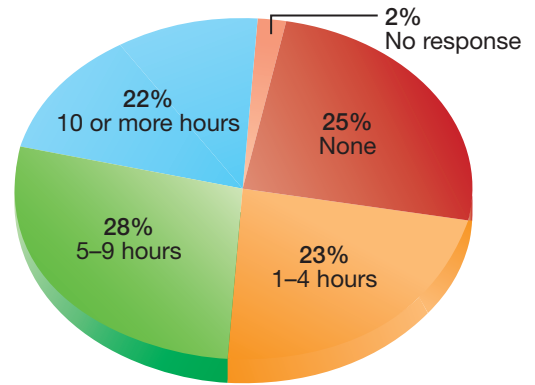
- $m\widehat{BC}$
- $m\widehat{EDB}$
- $m\widehat{CBE}$
- $m\widehat{CD}$



- Points  $T$  and  $R$  lie on  $\odot W$  so that  $WR = 12$  and  $m\angle TWR = 60$ . Find the length of  $\widehat{TR}$ .

- Application** 13. **SURVEYS** The graph shows the results of a survey of 1400 chief financial officers who were asked how many hours they spend working on the weekend. Determine the measurement of each angle of the graph. Round to the nearest degree.

**Executives Working on the Weekend**



Source: Accountemps

## Practice and Apply

### Homework Help

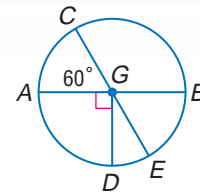
For Exercises	See Examples
14-23	1
24-39	2
40-43	3
44-45	4

### Extra Practice

See page 774.

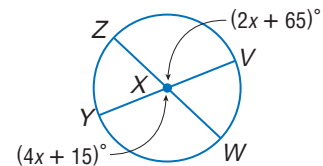
Find each measure.

- |                   |                   |
|-------------------|-------------------|
| 14. $m\angle CGB$ | 15. $m\angle BGE$ |
| 16. $m\angle AGD$ | 17. $m\angle DGE$ |
| 18. $m\angle CGD$ | 19. $m\angle AGE$ |



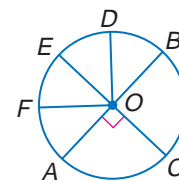
**ALGEBRA** Find each measure.

- |                   |                   |
|-------------------|-------------------|
| 20. $m\angle ZXV$ | 21. $m\angle YXW$ |
| 22. $m\angle ZXY$ | 23. $m\angle VXW$ |



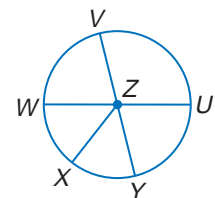
In  $\odot O$ ,  $\overline{EC}$  and  $\overline{AB}$  are diameters, and  $\angle BOD \cong \angle DOE \cong \angle EOF \cong \angle FOA$ . Find each measure.

- |                      |                      |
|----------------------|----------------------|
| 24. $m\widehat{BC}$  | 25. $m\widehat{AC}$  |
| 26. $m\widehat{AE}$  | 27. $m\widehat{EB}$  |
| 28. $m\widehat{ACB}$ | 29. $m\widehat{AD}$  |
| 30. $m\widehat{CBF}$ | 31. $m\widehat{ADC}$ |



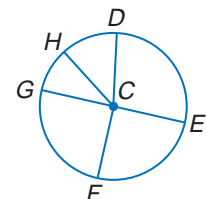
**ALGEBRA** In  $\odot Z$ ,  $\angle WZX \cong \angle XZY$ ,  $m\angle VZU = 4x$ ,  $m\angle UZY = 2x + 24$ , and  $\overline{VY}$  and  $\overline{WU}$  are diameters. Find each measure.

- |                      |                      |
|----------------------|----------------------|
| 32. $m\widehat{UY}$  | 33. $m\widehat{WV}$  |
| 34. $m\widehat{WX}$  | 35. $m\widehat{XY}$  |
| 36. $m\widehat{WUY}$ | 37. $m\widehat{YVW}$ |
| 38. $m\widehat{XVY}$ | 39. $m\widehat{WUX}$ |



The diameter of  $\odot C$  is 32 units long. Find the length of each arc for the given angle measure.

- |  |   |
|--|---|
| 40. $\widehat{DE}$ if $m\angle DCE = 100$  | 41. $\widehat{DHE}$ if $m\angle DCE = 90$ |
| 42. $\widehat{HDF}$ if $m\angle HCF = 125$ | 43. $\widehat{HD}$ if $m\angle DCH = 45$  |



**ONLINE MUSIC** For Exercises 44–46, refer to the table and use the following information.

A recent survey asked online users how many legally free music files they have collected. The results are shown in the table.

Free Music Downloads	
How many free music files have you collected?	
100 files or less	76%
101 to 500 files	16%
501 to 1000 files	5%
More than 1000 files	3%

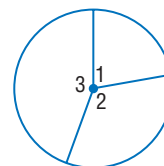
Source: QuickTake.com

- If you were to construct a circle graph of this information, how many degrees would be needed for each category?
- Describe the kind of arc associated with each category.
- Construct a circle graph for these data.

Determine whether each statement is *sometimes*, *always*, or *never* true.

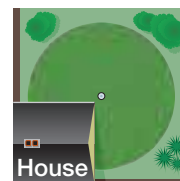
- The measure of a major arc is greater than 180.
- The central angle of a minor arc is an acute angle.
- The sum of the measures of the central angles of a circle depends on the measure of the radius.
- The semicircles of two congruent circles are congruent.

- CRITICAL THINKING** Central angles 1, 2, and 3 have measures in the ratio 2 : 3 : 4. Find the measure of each angle.



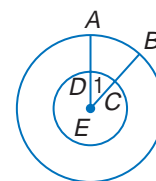
- CLOCKS** The hands of a clock form the same angle at various times of the day. For example, the angle formed at 2:00 is congruent to the angle formed at 10:00. If a clock has a diameter of 1 foot, what is the distance along the edge of the clock from the minute hand to the hour hand at 2:00?

- IRRIGATION** Some irrigation systems spray water in a circular pattern. You can adjust the nozzle to spray in certain directions. The nozzle in the diagram is set so it does not spray on the house. If the spray has a radius of 12 feet, what is the approximate length of the arc that the spray creates?



- PROOF** Write a proof of Theorem 10.1.

- CRITICAL THINKING** The circles at the right are concentric circles that both have point  $E$  as their center. If  $m\angle 1 = 42$ , determine whether  $\overline{AB} \cong \overline{CD}$ . Explain.



- WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**What kind of angles do the hands of a clock form?**

Include the following in your answer:

- the kind of angle formed by the hands of a clock, and
- several times of day when these angles are congruent.

### More About . . .

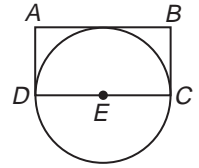


### Irrigation

In the Great Plains of the United States, farmers use center-pivot irrigation systems to water crops. New low-energy spray systems water circles of land that are thousands of feet in diameter with minimal water loss to evaporation from the spray.

Source: U.S. Geological Survey

57. Compare the circumference of circle  $E$  with the perimeter of rectangle  $ABCD$ . Which statement is true?



- (A) The perimeter of  $ABCD$  is greater than the circumference of circle  $E$ .  
 (B) The circumference of circle  $E$  is greater than the perimeter of  $ABCD$ .  
 (C) The perimeter of  $ABCD$  equals the circumference of circle  $E$ .  
 (D) There is not enough information to determine this comparison.
58. **SHORT RESPONSE** A circle is divided into three central angles that have measures in the ratio  $3 : 5 : 10$ . Find the measure of each angle.

## Maintain Your Skills

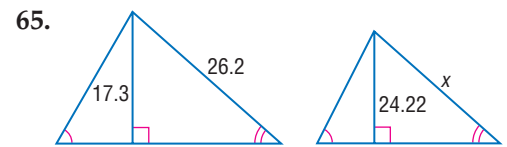
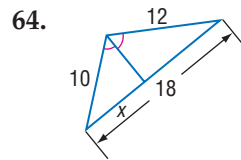
### Mixed Review

The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary. (Lesson 10-1)

59.  $r = 10, d = ? , C = ?$                       60.  $d = 13, r = ? , C = ?$   
 61.  $C = 28\pi, d = ? , r = ?$                       62.  $C = 75.4, d = ? , r = ?$

63. **SOCCER** Two soccer players kick the ball at the same time. One exerts a force of 72 newtons east. The other exerts a force of 45 newtons north. What is the magnitude to the nearest tenth and direction to the nearest degree of the resultant force on the soccer ball? (Lesson 9-6)

### ALGEBRA Find $x$ . (Lesson 6-5)



Find the exact distance between each point and line or pair of lines. (Lesson 3-6)

66. point  $Q(6, -2)$  and the line with the equation  $y - 7 = 0$   
 67. parallel lines with the equations  $y = x + 3$  and  $y = x - 4$   
 68. Angle  $A$  has a measure of  $57.5$ . Find the measures of the complement and supplement of  $\angle A$ . (Lesson 2-8)

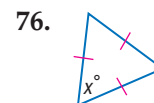
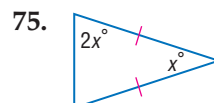
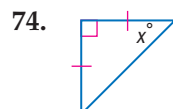
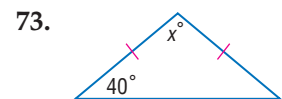
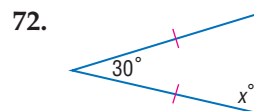
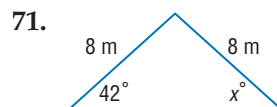
Use the following statement for Exercises 69 and 70.

If  $ABC$  is a triangle, then  $ABC$  has three sides. (Lesson 2-3)

69. Write the converse of the statement.  
 70. Determine the truth value of the statement and its converse.

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Find  $x$ . (To review **isosceles triangles**, see Lesson 4-6.)





# 10-3 Arcs and Chords

## What You'll Learn

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between chords and diameters.

## Vocabulary

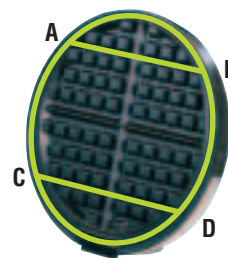
- inscribed
- circumscribed

## How do the grooves in a Belgian waffle iron model segments in a circle?

Waffle irons have grooves in each heated plate that result in the waffle pattern when the batter is cooked. One model of a Belgian waffle iron is round, and each groove is a chord of the circle.



**ARCS AND CHORDS** The endpoints of a chord are also endpoints of an arc. If you trace the waffle pattern on patty paper and fold along the diameter,  $\overline{AB}$  and  $\overline{CD}$  match exactly, as well as  $\widehat{AB}$  and  $\widehat{CD}$ . This suggests the following theorem.



## Theorem 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

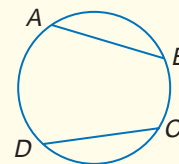
### Abbreviations:

In  $\odot X$ , 2 minor arcs are  $\cong$ , corr. chords are  $\cong$ .  
In  $\odot$ , 2 chords are  $\cong$ , corr. minor arcs are  $\cong$

### Examples

If  $\overline{AB} \cong \overline{CD}$ ,  
 $\widehat{AB} \cong \widehat{CD}$ .

If  $\widehat{AB} \cong \widehat{CD}$ ,  
 $\overline{AB} \cong \overline{CD}$ .



## Study Tip

### Reading Mathematics

Remember that the phrase *if and only if* means that the conclusion and the hypothesis can be switched and the statement is still true.

You will prove part 2 of Theorem 10.2 in Exercise 4.

## Example 1 Prove Theorems

**PROOF** Theorem 10.2 (part 1)

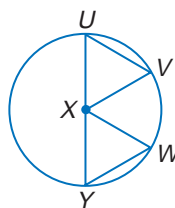
**Given:**  $\odot X$ ,  $\widehat{UV} \cong \widehat{YW}$

**Prove:**  $\overline{UV} \cong \overline{YW}$

**Proof:**

### Statements

1.  $\odot X$ ,  $\widehat{UV} \cong \widehat{YW}$
2.  $\angle UXV \cong \angle WXY$
3.  $\overline{UX} \cong \overline{XV} \cong \overline{XW} \cong \overline{XY}$
4.  $\triangle UXV \cong \triangle WXY$
5.  $\overline{UV} \cong \overline{YW}$



### Reasons

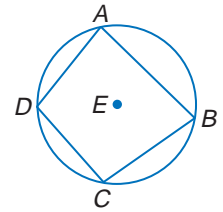
1. Given
2. If arcs are  $\cong$ , their corresponding central  $\angle$ s are  $\cong$ .
3. All radii of a circle are congruent.
4. SAS
5. CPCTC

## Study Tip

### Inscribed and Circumscribed

A circle can also be inscribed in a polygon, so that the polygon is circumscribed about the circle. You will learn about this in Lesson 10-5.

The chords of adjacent arcs can form a polygon. Quadrilateral  $ABCD$  is an **inscribed** polygon because all of its vertices lie on the circle. Circle  $E$  is **circumscribed** about the polygon because it contains all the vertices of the polygon.



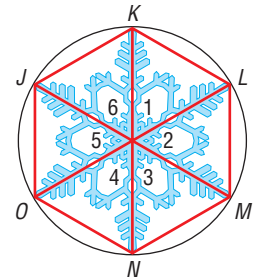
## Example 2 Inscribed Polygons

**SNOWFLAKES** The main veins of a snowflake create six congruent central angles. Determine whether the hexagon containing the flake is regular.

$$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 5 \cong \angle 6 \quad \text{Given}$$

$$\widehat{KL} \cong \widehat{LM} \cong \widehat{MN} \cong \widehat{NO} \cong \widehat{OJ} \cong \widehat{JK} \quad \text{If central } \angle\text{s are } \cong, \text{ corresponding arcs are } \cong.$$

$$\overline{KL} \cong \overline{LM} \cong \overline{MN} \cong \overline{NO} \cong \overline{OJ} \cong \overline{JK} \quad \text{In } \odot, 2 \text{ minor arcs } \cong, \text{ corr. chords are } \cong.$$



Because all the central angles are congruent, the measure of each angle is  $360 \div 6$  or  $60$ .

Let  $x$  be the measure of each base angle in the triangle containing  $\overline{KL}$ .

$$m\angle 1 + x + x = 180 \quad \text{Angle Sum Theorem}$$

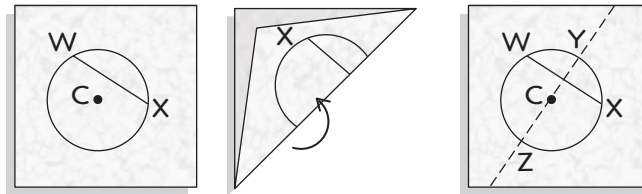
$$60 + 2x = 180 \quad \text{Substitution}$$

$$2x = 120 \quad \text{Subtract 60 from each side.}$$

$$x = 60 \quad \text{Divide each side by 2.}$$

This applies to each triangle in the figure, so each angle of the hexagon is  $2(60)$  or  $120$ . Thus the hexagon has all sides congruent and all vertex angles congruent.

**DIAMETERS AND CHORDS** Diameters that are perpendicular to chords create special segment and arc relationships. Suppose you draw circle  $C$  and one of its chords  $\overline{WX}$  on a piece of patty paper and fold the paper to construct the perpendicular bisector. You will find that the bisector also cuts  $\overline{WX}$  in half and passes through the center of the circle, making it contain a diameter.

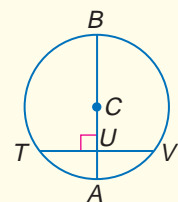


This is formally stated in the next theorem.

## Theorem 10.3

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

**Example:** If  $\overline{BA} \perp \overline{TV}$ , then  $\overline{UT} \cong \overline{UV}$  and  $\widehat{AT} \cong \widehat{AV}$ .



You will prove Theorem 10.3 in Exercise 36.

### Example 3 Radius Perpendicular to a Chord

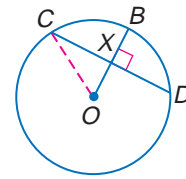
Circle  $O$  has a radius of 13 inches. Radius  $\overline{OB}$  is perpendicular to chord  $\overline{CD}$ , which is 24 inches long.

a. If  $m\widehat{CD} = 134$ , find  $m\widehat{CB}$ .

$\overline{OB}$  bisects  $\widehat{CD}$ , so  $m\widehat{CB} = \frac{1}{2}m\widehat{CD}$ .

$$m\widehat{CB} = \frac{1}{2}m\widehat{CD} \quad \text{Definition of arc bisector}$$

$$m\widehat{CB} = \frac{1}{2}(134) \text{ or } 67 \quad m\widehat{CD} = 134$$



b. Find  $OX$ .

Draw radius  $\overline{OC}$ .  $\triangle CXO$  is a right triangle.

$$CO = 13 \quad r = 13$$

$\overline{OB}$  bisects  $\widehat{CD}$ . A radius perpendicular to a chord bisects it.

$$CX = \frac{1}{2}(CD) \quad \text{Definition of segment bisector}$$

$$= \frac{1}{2}(24) \text{ or } 12 \quad CD = 24$$

Use the Pythagorean Theorem to find  $OX$ .

$$(CX)^2 + (OX)^2 = (CO)^2 \quad \text{Pythagorean Theorem}$$

$$12^2 + (OX)^2 = 13^2 \quad CX = 12, CO = 13$$

$$144 + (OX)^2 = 169 \quad \text{Simplify.}$$

$$(OX)^2 = 25 \quad \text{Subtract 144 from each side.}$$

$$OX = 5 \quad \text{Take the square root of each side.}$$

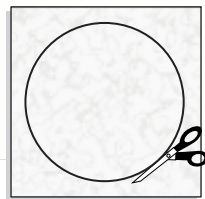
In the next activity, you will discover another property of congruent chords.

## Geometry Activity

### Congruent Chords and Distance

#### Model

**Step 1** Use a compass to draw a large circle on patty paper. Cut out the circle.



**Step 2** Fold the circle in half.



**Step 3** Without opening the circle, fold the edge of the circle so it does not intersect the first fold.



**Step 4** Unfold the circle and label as shown.



**Step 5** Fold the circle, laying point  $V$  onto  $T$  to bisect the chord. Open the circle and fold again to bisect  $\overline{WY}$ . Label as shown.



#### Analyze

1. What is the relationship between  $\overline{SU}$  and  $\overline{VT}$ ?  $\overline{SX}$  and  $\overline{WY}$ ?
2. Use a centimeter ruler to measure  $\overline{VT}$ ,  $\overline{WY}$ ,  $\overline{SU}$ , and  $\overline{SX}$ . What do you find?
3. **Make a conjecture** about the distance that two chords are from the center when they are congruent.

The Geometry Activity suggests the following theorem.

### Theorem 10.4

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

You will prove Theorem 10.4 in Exercises 37 and 38.

#### Example 4 Chords Equidistant from Center

Chords  $\overline{AC}$  and  $\overline{DF}$  are equidistant from the center. If the radius of  $\odot G$  is 26, find  $AC$  and  $DE$ .

$\overline{AC}$  and  $\overline{DF}$  are equidistant from  $G$ , so  $\overline{AC} \cong \overline{DF}$ .

Draw  $\overline{AG}$  and  $\overline{GF}$  to form two right triangles. Use the Pythagorean Theorem.

$$(AB)^2 + (BG)^2 = (AG)^2 \quad \text{Pythagorean Theorem}$$

$$(AB)^2 + 10^2 = 26^2 \quad BG = 10, AG = 26$$

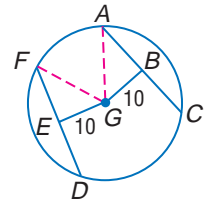
$$(AB)^2 + 100 = 676 \quad \text{Simplify.}$$

$$(AB)^2 = 576 \quad \text{Subtract 100 from each side.}$$

$$AB = 24 \quad \text{Take the square root of each side.}$$

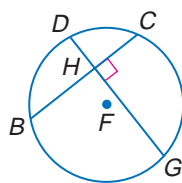
$$AB = \frac{1}{2}(AC), \text{ so } AC = 2(24) \text{ or } 48.$$

$$\overline{AC} \cong \overline{DF}, \text{ so } DF \text{ also equals } 48. DE = \frac{1}{2}DF, \text{ so } DE = \frac{1}{2}(48) \text{ or } 24.$$



## Check for Understanding

- Concept Check**
1. Explain the difference between an inscribed polygon and a circumscribed circle.
  2. **OPEN ENDED** Construct a circle and inscribe any polygon. Draw the radii and use a protractor to determine whether any sides of the polygon are congruent.
  3. **FIND THE ERROR** Lucinda and Tokei are writing conclusions about the chords in  $\odot F$ . Who is correct? Explain your reasoning.



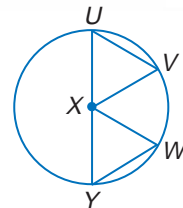
Lucinda  
Because  $\overline{DG} \perp \overline{BC}$ ,  
 $\angle DHB \cong \angle DHC \cong$   
 $\angle CHG \cong \angle BHG$ ,  
and  $\overline{DG}$  bisects  $\overline{BC}$ .

Tokei  
 $\overline{DG} \perp \overline{BC}$ , but  $\overline{DG}$   
does not bisect  $\overline{BC}$   
because it is not  
a diameter.

- Guided Practice**
4. **PROOF** Prove part 2 of Theorem 10.2.

**Given:**  $\odot X, \overline{UV} \cong \overline{YW}$

**Prove:**  $\widehat{UV} \cong \widehat{YW}$

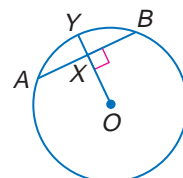


Circle  $O$  has a radius of 10,  $AB = 10$ , and  $m\widehat{AB} = 60$ . Find each measure.

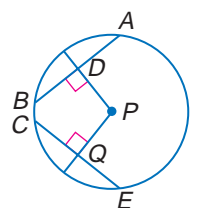
5.  $m\widehat{AY}$
6.  $AX$
7.  $OX$

In  $\odot P$ ,  $PD = 10$ ,  $PQ = 10$ , and  $QE = 20$ . Find each measure.

8.  $AB$
9.  $PE$



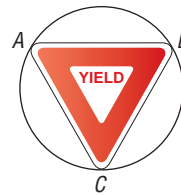
Exercises 5–7



Exercises 8–9



- Application** 10. **TRAFFIC SIGNS** A yield sign is an equilateral triangle. Find the measure of each arc of the circle circumscribed about the yield sign.



## Practice and Apply

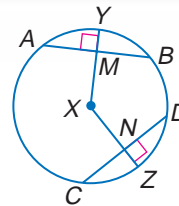
### Homework Help

For Exercises	See Examples
11–22	3
23–25	2
26–33	4
36–38	1

**Extra Practice**  
See page 774.

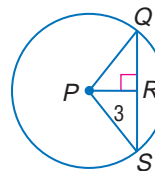
In  $\odot X$ ,  $AB = 30$ ,  $CD = 30$ , and  $m\widehat{CZ} = 40$ . Find each measure.

- |                     |                     |
|---------------------|---------------------|
| 11. $AM$            | 12. $MB$            |
| 13. $CN$            | 14. $ND$            |
| 15. $m\widehat{DZ}$ | 16. $m\widehat{CD}$ |
| 17. $m\widehat{AB}$ | 18. $m\widehat{YB}$ |

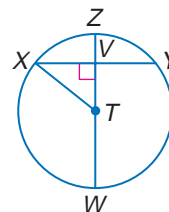


The radius of  $\odot P$  is 5 and  $PR = 3$ . Find each measure.

- |          |          |
|----------|----------|
| 19. $QR$ | 20. $QS$ |
|----------|----------|



Exercises 19–20



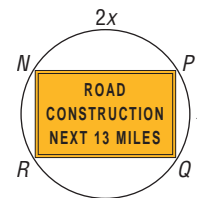
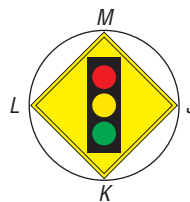
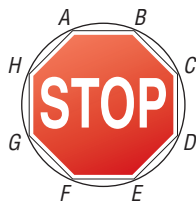
Exercises 21–22

In  $\odot T$ ,  $ZV = 1$ , and  $TW = 13$ . Find each measure.

- |          |          |
|----------|----------|
| 21. $XV$ | 22. $XY$ |
|----------|----------|

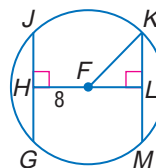
**TRAFFIC SIGNS** Determine the measure of each arc of the circle circumscribed about the traffic sign.

- |                     |            |               |
|---------------------|------------|---------------|
| 23. regular octagon | 24. square | 25. rectangle |
|---------------------|------------|---------------|



In  $\odot E$ ,  $\overline{FH} \cong \overline{FL}$  and  $FK = 17$ . Find each measure.

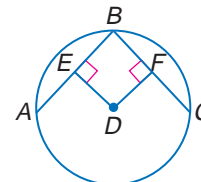
- |          |          |
|----------|----------|
| 26. $LK$ | 27. $KM$ |
| 28. $JG$ | 29. $JH$ |



Exercises 26–29

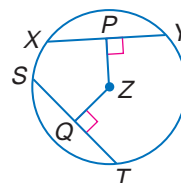
In  $\odot D$ ,  $CF = 8$ ,  $DE = FD$ , and  $DC = 10$ . Find each measure.

- |          |          |
|----------|----------|
| 30. $FB$ | 31. $BC$ |
| 32. $AB$ | 33. $ED$ |



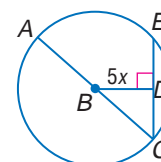
Exercises 30–33

34. **ALGEBRA** In  $\odot Z$ ,  $PZ = ZQ$ ,  $XY = 4a - 5$ , and  $ST = -5a + 13$ . Find  $SQ$ .



Exercise 34

35. **ALGEBRA** In  $\odot B$ , the diameter is 20 units long, and  $m\angle ACE = 45$ . Find  $x$ .



Exercise 35

More About . . .



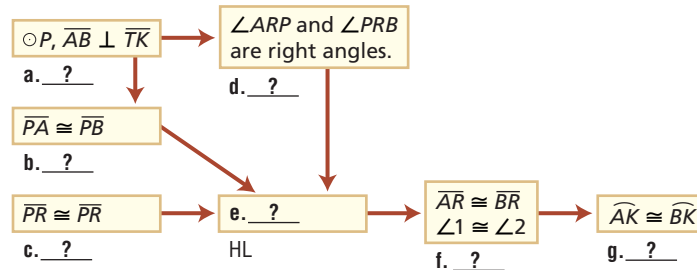
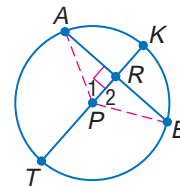
Sayings

Many everyday sayings or expressions have a historical origin. The square peg comment is attributed to Sydney Smith (1771–1845), a witty British lecturer, who said “Trying to get those two together is like trying to put a square peg in a round hole.”

36. **PROOF** Copy and complete the flow proof of Theorem 10.3.

**Given:**  $\odot P, \overline{AB} \perp \overline{TK}$

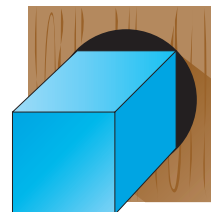
**Prove:**  $\overline{AR} \cong \overline{BR}, \widehat{AK} \cong \widehat{BK}$



- PROOF** Write a proof for each part of Theorem 10.4.

37. In a circle, if two chords are equidistant from the center, then they are congruent.  
38. In a circle, if two chords are congruent, then they are equidistant from the center.

39. **SAYINGS** An old adage states that “You can’t fit a square peg in a round hole.” Actually, you can, it just won’t fill the hole. If a hole is 4 inches in diameter, what is the approximate width of the largest square peg that fits in the round hole?



For Exercises 40–43, draw and label a figure. Then solve.

40. The radius of a circle is 34 meters long, and a chord of the circle is 60 meters long. How far is the chord from the center of the circle?  
41. The diameter of a circle is 60 inches, and a chord of the circle is 48 inches long. How far is the chord from the center of the circle?  
42. A chord of a circle is 48 centimeters long and is 10 centimeters from the center of the circle. Find the radius.  
43. A diameter of a circle is 32 yards long. A chord is 11 yards from the center. How long is the chord?

44. **CARPENTRY** Mr. Ortega wants to drill a hole in the center of a round picnic table for an umbrella pole. To locate the center of the circle, he draws two chords of the circle and uses a ruler to find the midpoint for each chord. Then he uses a framing square to draw a line perpendicular to each chord at its midpoint. Explain how this process locates the center of the tabletop.



45. **CRITICAL THINKING** A diameter of  $\odot P$  has endpoints  $A$  and  $B$ . Radius  $\overline{PQ}$  is perpendicular to  $\overline{AB}$ . Chord  $\overline{DE}$  bisects  $\overline{PQ}$  and is parallel to  $\overline{AB}$ . Does  $DE = \frac{1}{2}(AB)$ ? Explain.

Study Tip

Finding the Center of a Circle

The process Mr. Ortega used can be done by construction and is often called *locating the center of a circle*.

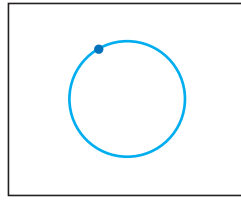


**CONSTRUCTION**

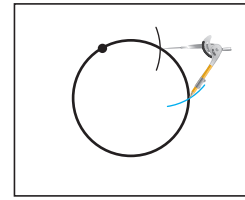
Use the following steps for each construction in Exercises

46 and 47.

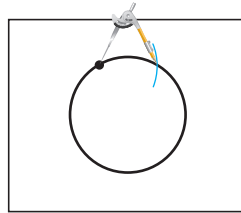
- 1 Construct a circle, and place a point on the circle.



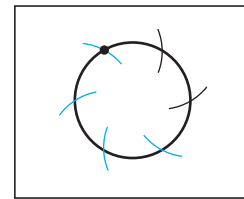
- 3 Using the same radius, place the compass on the intersection and draw another small arc to intercept the circle.



- 2 Using the same radius, place the compass on the point and draw a small arc to intercept the circle.



- 4 Continue the process in Step 3 until you return to the original point.

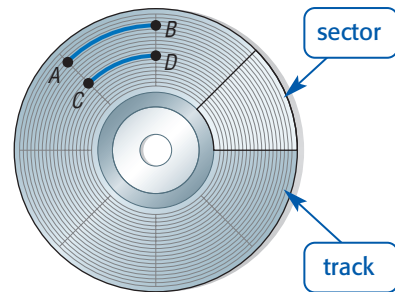


46. Connect the intersections with chords of the circle. What type of figure is formed? Verify your conjecture.
47. Repeat the construction. Connect every other intersection with chords of the circle. What type of figure is formed? Verify your conjecture.

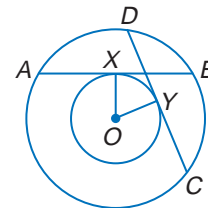
**COMPUTERS** For Exercises 48 and 49, use the following information.

The hard drive of a computer contains platters divided into tracks, which are defined by concentric circles, and sectors, which are defined by radii of the circles.

48. In the diagram of a hard drive platter at the right, what is the relationship between  $m\widehat{AB}$  and  $m\widehat{CD}$ ?
49. Are  $\widehat{AB}$  and  $\widehat{CD}$  congruent? Explain.



50. **CRITICAL THINKING** The figure shows two concentric circles with  $\overline{OX} \perp \overline{AB}$  and  $\overline{OY} \perp \overline{CD}$ . Write a statement relating  $\widehat{AB}$  and  $\widehat{CD}$ . Verify your reasoning.



51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How do the grooves in a Belgian waffle iron model segments in a circle?**

Include the following in your answer:

- a description of how you might find the length of a groove without directly measuring it, and
- a sketch with measurements for a waffle iron that is 8 inches wide.

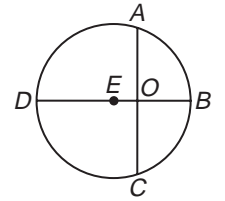


**Standardized  
Test Practice**

(A) (B) (C) (D)

52. Refer to the figure. Which of the following statements is true?

- I.  $\overline{DB}$  bisects  $\overline{AC}$ . II.  $\overline{AC}$  bisects  $\overline{DB}$ . III.  $OA = OC$
- (A) I and II (B) II and III  
(C) I and III (D) I, II, and III



53. **SHORT RESPONSE** According to the 2000 census, the population of Bridgeworth was 204 thousand, and the population of Sutterly was 216 thousand. If the population of each city increased by exactly 20% ten years later, how many more people will live in Sutterly than in Bridgeworth in 2010?

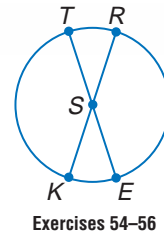
**Maintain Your Skills**

**Mixed Review** In  $\odot S$ ,  $m\angle TSR = 42$ . Find each measure.  
(Lesson 10-2)

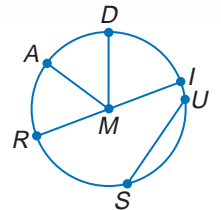
54.  $m\widehat{KT}$       55.  $m\widehat{ERT}$       56.  $m\widehat{KRT}$

Refer to  $\odot M$ . (Lesson 10-1)

57. Name a chord that is not a diameter.  
58. If  $MD = 7$ , find  $RI$ .  
59. Name congruent segments in  $\odot M$ .



Exercises 54-56



Exercises 57-59

**Getting Ready for  
the Next Lesson**

**PREREQUISITE SKILL** Solve each equation.

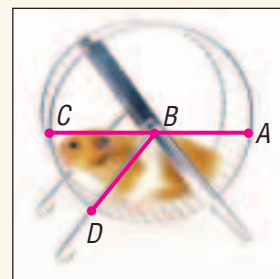
(To review solving equations, see pages 742 and 743.)

60.  $\frac{1}{2}x = 120$       61.  $\frac{1}{2}x = 25$       62.  $2x = \frac{1}{2}(45 + 35)$   
63.  $3x = \frac{1}{2}(120 - 60)$       64.  $45 = \frac{1}{2}(4x + 30)$       65.  $90 = \frac{1}{2}(6x + 3x)$

**Practice Quiz 1**

**Lessons 10-1 through 10-3**

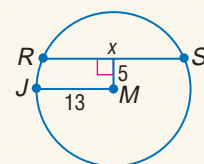
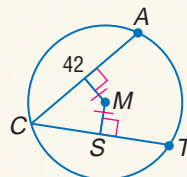
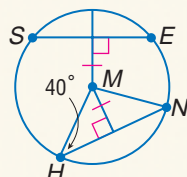
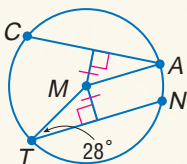
**PETS** For Exercises 1-6, refer to the front circular edge of the hamster wheel shown at the right. (Lessons 10-1 and 10-2)



- Name three radii of the wheel.
- If  $BD = 3x$  and  $CB = 7x - 3$ , find  $AC$ .
- If  $m\angle CBD = 85$ , find  $m\widehat{AD}$ .
- If  $r = 3$  inches, find the circumference of circle  $B$  to the nearest tenth of an inch.
- There are 40 equally-spaced rungs on the wheel. What is the degree measure of an arc connecting two consecutive rungs?
- What is the length of  $\widehat{CAD}$  to the nearest tenth if  $m\angle ABD = 150$  and  $r = 3$ ?

Find each measure. (Lesson 10-3)

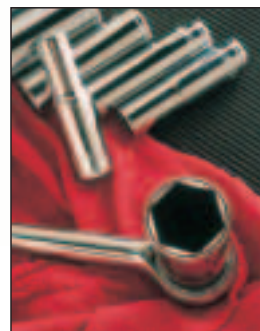
7.  $m\angle CAM$       8.  $m\widehat{ES}$       9.  $SC$       10.  $x$



# 10-4 Inscribed Angles

## What You'll Learn

- Find measures of inscribed angles.
- Find measures of angles of inscribed polygons.



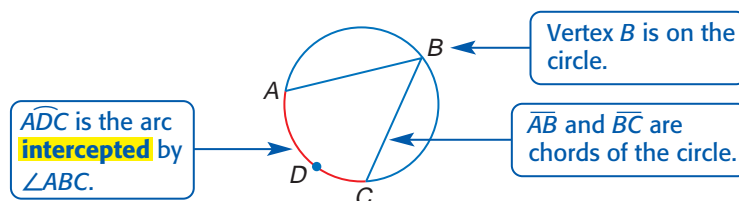
## Vocabulary

- intercepted

## How is a socket like an inscribed polygon?

A socket is a tool that comes in varying diameters. It is used to tighten or unscrew nuts. The “hole” in the socket is a hexagon cast in a metal cylinder, which attaches to a wrench mechanism for leverage.

**INSCRIBED ANGLES** In Lesson 10-3, you learned that a polygon that has its vertices on a circle is called an inscribed polygon. Likewise, an *inscribed angle* is an angle that has its vertex on the circle and its sides contained in chords of the circle.

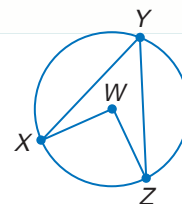


## Geometry Activity

### Measure of Inscribed Angles

#### Model

- Use a compass to draw a circle and label the center  $W$ .
- Draw an inscribed angle and label it  $XYZ$ .
- Draw  $\overline{WX}$  and  $\overline{WZ}$ .



#### Analyze

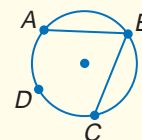
1. Measure  $\angle XYZ$  and  $\angle XWZ$ .
2. Find  $m\widehat{XZ}$  and compare it with  $m\angle XYZ$ .
3. **Make a conjecture** about the relationship of the measure of an inscribed angle and the measure of its intercepted arc.

This activity suggests the following theorem.

## Theorem 10.5

**Inscribed Angle Theorem** If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

**Example:**  $m\angle ABC = \frac{1}{2}(m\widehat{ADC})$  or  $2(m\angle ABC) = m\widehat{ADC}$





To prove Theorem 10.5, you must consider three cases.

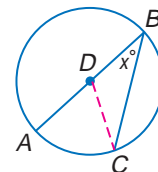
	Case 1	Case 2	Case 3
Model of Angle Inscribed in $\odot O$			
Location of center of circle	on a side of the angle	in the interior of the angle	in the exterior of the angle

### Proof Theorem 10.5 (Case 1)

**Given:**  $\angle ABC$  inscribed in  $\odot D$  and  $\overline{AB}$  is a diameter.

**Prove:**  $m\angle ABC = \frac{1}{2}m\widehat{AC}$

Draw  $\overline{DC}$  and let  $m\angle B = x$ .



**Proof:**

Since  $\overline{DB}$  and  $\overline{DC}$  are congruent radii,  $\triangle BDC$  is isosceles and  $\angle B \cong \angle C$ . Thus,  $m\angle B = m\angle C = x$ . By the Exterior Angle Theorem,  $m\angle ADC = m\angle B + m\angle C$ . So  $m\angle ADC = 2x$ . From the definition of arc measure, we know that  $m\widehat{AC} = m\angle ADC$  or  $2x$ . Comparing  $m\widehat{AC}$  and  $m\angle ABC$ , we see that  $m\widehat{AC} = 2(m\angle ABC)$  or that  $m\angle ABC = \frac{1}{2}m\widehat{AC}$ .

You will prove Cases 2 and 3 of Theorem 10.5 in Exercises 35 and 36.

### Study Tip

#### Using Variables

You can also assign a variable to an unknown measure. So, if you let  $m\widehat{AD} = x$ , the second equation becomes  $140 + 100 + x + x = 360$ , or  $240 + 2x = 360$ . This last equation may seem simpler to solve.

### Example 1 Measures of Inscribed Angles

In  $\odot O$ ,  $m\widehat{AB} = 140$ ,  $m\widehat{BC} = 100$ , and  $m\widehat{AD} = m\widehat{DC}$ .

Find the measures of the numbered angles.

First determine  $m\widehat{DC}$  and  $m\widehat{AD}$ .

$$m\widehat{AB} + m\widehat{BC} + m\widehat{DC} + m\widehat{AD} = 360 \quad \text{Arc Addition Theorem}$$

$$140 + 100 + m\widehat{DC} + m\widehat{DC} = 360 \quad \begin{array}{l} m\widehat{AB} = 140, m\widehat{BC} = 100, \\ m\widehat{DC} = m\widehat{AD} \end{array}$$

$$240 + 2(m\widehat{DC}) = 360 \quad \text{Simplify.}$$

$$2(m\widehat{DC}) = 120 \quad \text{Subtract 240 from each side.}$$

$$m\widehat{DC} = 60 \quad \text{Divide each side by 2.}$$

So,  $m\widehat{DC} = 60$  and  $m\widehat{AD} = 60$ .

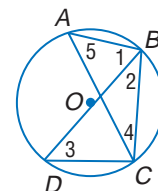
$$\begin{aligned} m\angle 1 &= \frac{1}{2}m\widehat{AD} \\ &= \frac{1}{2}(60) \text{ or } 30 \end{aligned}$$

$$\begin{aligned} m\angle 2 &= \frac{1}{2}m\widehat{DC} \\ &= \frac{1}{2}(60) \text{ or } 30 \end{aligned}$$

$$\begin{aligned} m\angle 3 &= \frac{1}{2}m\widehat{BC} \\ &= \frac{1}{2}(100) \text{ or } 50 \end{aligned}$$

$$\begin{aligned} m\angle 4 &= \frac{1}{2}m\widehat{AB} \\ &= \frac{1}{2}(140) \text{ or } 70 \end{aligned}$$

$$\begin{aligned} m\angle 5 &= \frac{1}{2}m\widehat{BC} \\ &= \frac{1}{2}(100) \text{ or } 50 \end{aligned}$$



In Example 1, note that  $\angle 3$  and  $\angle 5$  intercept the same arc and are congruent.

### Theorem 10.6

If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

**Abbreviations:**

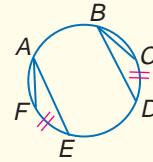
Inscribed  $\angle$ s of  $\cong$  arcs are  $\cong$ .

Inscribed  $\angle$ s of same arc are  $\cong$ .

**Examples:**



$$\angle DAC \cong \angle DBC$$



$$\angle FAE \cong \angle CBD$$

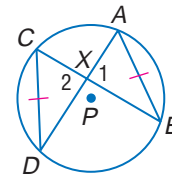
You will prove Theorem 10.6 in Exercise 37.

### Example 2 Proofs with Inscribed Angles

**Given:**  $\odot P$  with  $\overline{CD} \cong \overline{AB}$

**Prove:**  $\triangle AXB \cong \triangle CXD$

**Proof:**



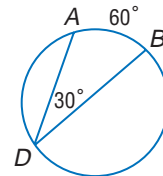
Statements	Reasons
1. $\angle DAB$ intercepts $\overline{DB}$ . $\angle DCB$ intercepts $\overline{DB}$ .	1. Definition of intercepted arc
2. $\angle DAB \cong \angle DCB$	2. Inscribed $\angle$ s of same arc are $\cong$ .
3. $\angle 1 \cong \angle 2$	3. Vertical $\angle$ s are $\cong$ .
4. $\overline{CD} \cong \overline{AB}$	4. Given
5. $\triangle AXB \cong \triangle CXD$	5. AAS

You can also use the measure of an inscribed angle to determine probability of a point lying on an arc.

### Example 3 Inscribed Arcs and Probability

**PROBABILITY** Points  $A$  and  $B$  are on a circle so that  $m\widehat{AB} = 60$ . Suppose point  $D$  is randomly located on the same circle so that it does not coincide with  $A$  or  $B$ . What is the probability that  $m\angle ADB = 30$ ?

Since the angle measure is half the arc measure, inscribed  $\angle ADB$  must intercept  $\widehat{AB}$ , so  $D$  must lie on major arc  $AB$ . Draw a figure and label any information you know.



$$\begin{aligned} m\widehat{BDA} &= 360 - m\widehat{AB} \\ &= 360 - 60 \text{ or } 300 \end{aligned}$$

Since  $\angle ADB$  must intercept  $\widehat{AB}$ , the probability that  $m\angle ADB = 30$  is the same as the probability of  $D$  being contained in  $\widehat{BDA}$ .

The probability that  $D$  is located on  $\widehat{ADB}$  is  $\frac{5}{6}$ . So, the probability that  $m\angle ADB = 30$  is also  $\frac{5}{6}$ .

#### Study Tip

#### Eliminate the Possibilities

Think about what would be true if  $D$  was on minor arc  $\widehat{AB}$ . Then  $\angle ADB$  would intercept the major arc. Thus,  $m\angle ADB$  would be half of 300 or 150. This is not the desired angle measure in the problem, so you can eliminate the possibility that  $D$  can lie on  $\widehat{AB}$ .

## Study Tip

### Inscribed Polygons

Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

## ANGLES OF INSCRIBED POLYGONS

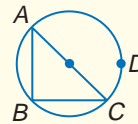
An inscribed triangle with a side that is a diameter is a special type of triangle.

An inscribed triangle with a side that is a diameter is a special type of triangle.

### Theorem 10.7

If an inscribed angle intercepts a semicircle, the angle is a right angle.

**Example:**  $\widehat{ADC}$  is a semicircle, so  $m\angle ABC = 90$ .



You will prove Theorem 10.7 in Exercise 38.

### Example 4 Angles of an Inscribed Triangle

**ALGEBRA** Triangles  $ABD$  and  $ADE$  are inscribed in  $\odot F$  with  $\widehat{AB} \cong \widehat{BD}$ . Find the measure of each numbered angle if  $m\angle 1 = 12x - 8$  and  $m\angle 2 = 3x + 8$ .

$\angle AED$  is a right angle because  $\widehat{AED}$  is a semicircle.

$$m\angle 1 + m\angle 2 + m\angle AED = 180 \quad \text{Angle Sum Theorem}$$

$$(12x - 8) + (3x + 8) + 90 = 180 \quad m\angle 1 = 12x - 8, m\angle 2 = 3x + 8, m\angle AED = 90$$

$$15x + 90 = 180 \quad \text{Simplify.}$$

$$15x = 90 \quad \text{Subtract 90 from each side.}$$

$$x = 6 \quad \text{Divide each side by 15.}$$

Use the value of  $x$  to find the measures of  $\angle 1$  and  $\angle 2$ .

$$m\angle 1 = 12x - 8 \quad \text{Given} \qquad m\angle 2 = 3x + 8 \quad \text{Given}$$

$$= 12(6) - 8 \quad x = 6 \qquad = 3(6) + 8 \quad x = 6$$

$$= 64 \quad \text{Simplify.} \qquad = 26 \quad \text{Simplify.}$$

Angle  $ABD$  is a right angle because it intercepts a semicircle.

Because  $\widehat{AB} \cong \widehat{BD}$ ,  $\widehat{AB} \cong \widehat{BD}$ , which leads to  $\angle 3 \cong \angle 4$ . Thus,  $m\angle 3 = m\angle 4$ .

$$m\angle 3 + m\angle 4 + m\angle ABD = 180 \quad \text{Angle Sum Theorem}$$

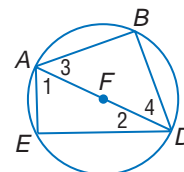
$$m\angle 3 + m\angle 3 + 90 = 180 \quad m\angle 3 = m\angle 4, m\angle ABD = 90$$

$$2(m\angle 3) + 90 = 180 \quad \text{Simplify.}$$

$$2(m\angle 3) = 90 \quad \text{Subtract 90 from each side.}$$

$$m\angle 3 = 45 \quad \text{Divide each side by 2.}$$

Since  $m\angle 3 = m\angle 4$ ,  $m\angle 4 = 45$ .



### Example 5 Angles of an Inscribed Quadrilateral

Quadrilateral  $ABCD$  is inscribed in  $\odot P$ . If  $m\angle B = 80$  and  $m\angle C = 40$ , find  $m\angle A$  and  $m\angle D$ .

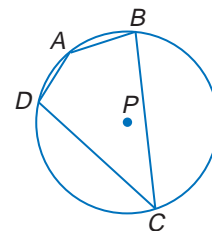
Draw a sketch of this situation.

To find  $m\angle A$ , we need to know  $m\widehat{BCD}$ .

To find  $m\widehat{BCD}$ , first find  $m\widehat{DAB}$ .

$$m\widehat{DAB} = 2(m\angle C) \quad \text{Inscribed Angle Theorem}$$

$$= 2(40) \text{ or } 80 \quad m\angle C = 40$$



(continued on the next page)

$$\begin{aligned}
m\widehat{BCD} + m\widehat{DAB} &= 360 && \text{Sum of angles in circle} = 360 \\
m\widehat{BCD} + 80 &= 360 && m\widehat{DAB} = 80 \\
m\widehat{BCD} &= 280 && \text{Subtract 80 from each side.} \\
m\widehat{BCD} &= 2(m\angle A) && \text{Inscribed Angle Theorem} \\
280 &= 2(m\angle A) && \text{Substitution} \\
140 &= m\angle A && \text{Divide each side by 2.}
\end{aligned}$$

To find  $m\angle D$ , we need to know  $m\widehat{ABC}$ , but first we must find  $m\widehat{ADC}$ .

$$\begin{aligned}
m\widehat{ADC} &= 2(m\angle B) && \text{Inscribed Angle Theorem} \\
m\widehat{ADC} &= 2(80) \text{ or } 160 && m\angle B = 80 \\
m\widehat{ABC} + m\widehat{ADC} &= 360 && \text{Sum of angles in circle} = 360 \\
m\widehat{ABC} + 160 &= 360 && m\widehat{ADC} = 160 \\
m\widehat{ABC} &= 200 && \text{Subtract 160 from each side.} \\
m\widehat{ABC} &= 2(m\angle D) && \text{Inscribed Angle Theorem} \\
200 &= 2(m\angle D) && \text{Substitution} \\
100 &= m\angle D && \text{Divide each side by 2.}
\end{aligned}$$

In Example 5, note that the opposite angles of the quadrilateral are supplementary. This is stated in Theorem 10.8 and can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

### Theorem 10.8

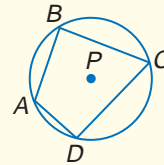
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

**Example:**

Quadrilateral  $ABCD$  is inscribed in  $\odot P$ .

$\angle A$  and  $\angle C$  are supplementary.

$\angle B$  and  $\angle D$  are supplementary.

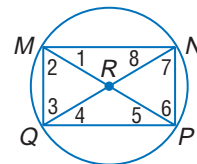


You will prove this theorem in Exercise 39.

## Check for Understanding

- Concept Check**
- OPEN ENDED** Draw a counterexample of an inscribed trapezoid. If possible, include at least one angle that is an inscribed angle.
  - Compare and contrast** an inscribed angle and a central angle that intercepts the same arc.

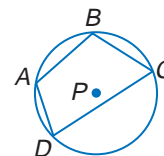
- Guided Practice**
- In  $\odot R$ ,  $m\widehat{MN} = 120$  and  $m\widehat{MQ} = 60$ . Find the measure of each numbered angle.



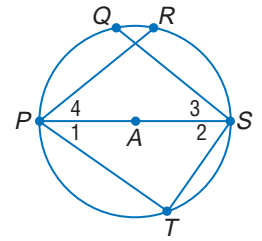
- PROOF** Write a paragraph proof.  
**Given:** Quadrilateral  $ABCD$  is inscribed in  $\odot P$ .

$$m\angle C = \frac{1}{2}m\angle B$$

**Prove:**  $m\widehat{CDA} = 2(m\widehat{DAB})$



5. **ALGEBRA** In  $\odot A$  at the right,  $\widehat{PQ} \cong \widehat{RS}$ . Find the measure of each numbered angle if  $m\angle 1 = 6x + 11$ ,  $m\angle 2 = 9x + 19$ ,  $m\angle 3 = 4y - 25$ , and  $m\angle 4 = 3y - 9$ .
6. Suppose quadrilateral  $VWXY$  is inscribed in  $\odot C$ . If  $m\angle X = 28$  and  $m\angle W = 110$ , find  $m\angle V$  and  $m\angle Y$ .



- Application** 7. **PROBABILITY** Points  $X$  and  $Y$  are endpoints of a diameter of  $\odot W$ . Point  $Z$  is another point on the circle. Find the probability that  $\angle XZY$  is a right angle.

## Practice and Apply

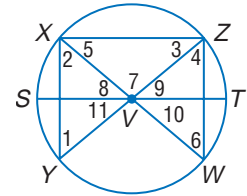
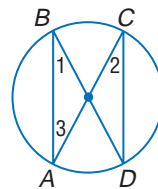
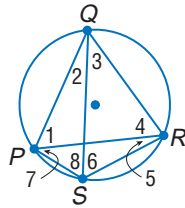
### Homework Help

For Exercises	See Examples
8–10	1
11–12, 35–39	2
13–17	4
18–21, 26–29	5
31–34	3

**Extra Practice**  
See page 774.

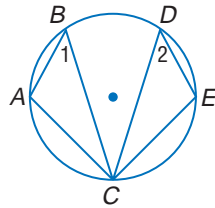
Find the measure of each numbered angle for each figure.

8.  $\widehat{PQ} \cong \widehat{RQ}$ ,  $m\widehat{PS} = 45$ , and  $m\widehat{SR} = 75$
9.  $m\angle BDC = 25$ ,  $m\widehat{AB} = 120$ , and  $m\widehat{CD} = 130$
10.  $m\widehat{XZ} = 100$ ,  $\overline{XY} \perp \overline{ST}$ , and  $\overline{ZW} \perp \overline{ST}$

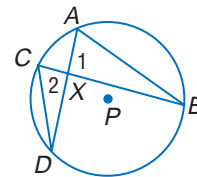


**PROOF** Write a two-column proof.

11. **Given:**  $\widehat{AB} \cong \widehat{DE}$ ,  $\widehat{AC} \cong \widehat{CE}$   
**Prove:**  $\triangle ABC \cong \triangle EDC$

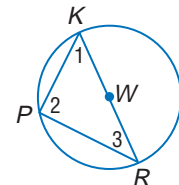
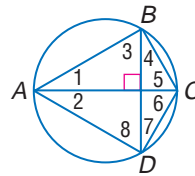
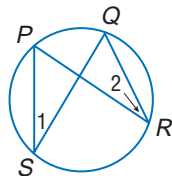


12. **Given:**  $\odot P$   
**Prove:**  $\triangle AXB \sim \triangle CXD$

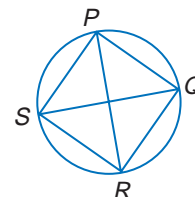


**ALGEBRA** Find the measure of each numbered angle for each figure.

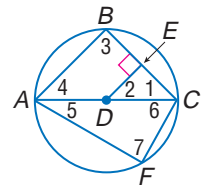
13.  $m\angle 1 = x$ ,  $m\angle 2 = 2x - 13$
14.  $m\widehat{AB} = 120$
15.  $m\angle R = \frac{1}{3}x + 5$ ,  
 $m\angle K = \frac{1}{2}x$



16.  $PQRS$  is a rhombus inscribed in a circle. Find  $m\angle QRP$  and  $m\widehat{SP}$ .
17. In  $\odot D$ ,  $\widehat{DE} \cong \widehat{EC}$ ,  $m\widehat{CF} = 60$ , and  $\overline{DE} \perp \overline{EC}$ . Find  $m\angle 4$ ,  $m\angle 5$ , and  $m\widehat{AF}$ .



Exercise 16

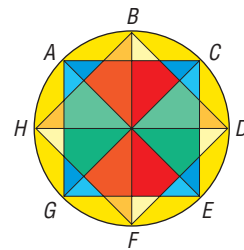


Exercise 17





**STAINED GLASS** In the stained glass window design, all of the small arcs around the circle are congruent. Suppose the center of the circle is point  $O$ .



40. What is the measure of each of the small arcs?  
 41. What kind of figure is  $\triangle AOC$ ? Explain.  
 42. What kind of figure is quadrilateral  $BDFH$ ? Explain.  
 43. What kind of figure is quadrilateral  $ACEG$ ? Explain.
44. **CRITICAL THINKING** A trapezoid  $ABCD$  is inscribed in  $\odot O$ . Explain how you can verify that  $ABCD$  must be an isosceles trapezoid.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is a socket like an inscribed polygon?**

Include the following in your answer:

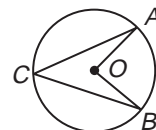
- a definition of an inscribed polygon, and
- the side length of a regular hexagon inscribed in a circle  $\frac{3}{4}$  inch wide.

**Standardized Test Practice**



46. What is the ratio of the measure of  $\angle ACB$  to the measure of  $\angle AOB$ ?

- (A) 1 : 1                      (B) 2 : 1  
 (C) 1 : 2                      (D) not enough information

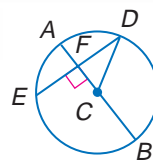


47. **GRID IN** The daily newspaper always follows a particular format. Each even-numbered page contains six articles, and each odd-numbered page contains seven articles. If today's paper has 36 pages, how many articles does it contain?

**Maintain Your Skills**

**Mixed Review** Find each measure. (Lesson 10-3)

48. If  $AB = 60$  and  $DE = 48$ , find  $CF$ .  
 49. If  $AB = 32$  and  $FC = 11$ , find  $FE$ .  
 50. If  $DE = 60$  and  $FC = 16$ , find  $AB$ .



Points  $Q$  and  $R$  lie on  $\odot P$ . Find the length of  $\widehat{QR}$  for the given radius and angle measure. (Lesson 10-2)

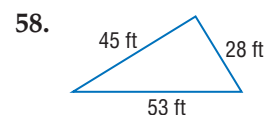
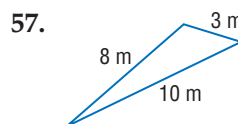
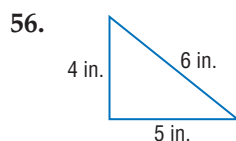
51.  $PR = 12$ , and  $m\angle QPR = 60$                       52.  $m\angle QPR = 90$ ,  $PR = 16$

Complete each sentence with *sometimes*, *always*, or *never*. (Lesson 4-1)

53. Equilateral triangles are   ?   isosceles.  
 54. Acute triangles are   ?   equilateral.  
 55. Obtuse triangles are   ?   scalene.

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Determine whether each figure is a right triangle. (To review the *Pythagorean Theorem*, see Lesson 7-2.)



# 10-5 Tangents

## What You'll Learn

- Use properties of tangents.
- Solve problems involving circumscribed polygons.

## Vocabulary

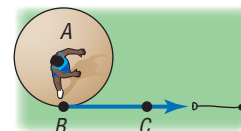
- tangent
- point of tangency

## How are tangents related to track and field events?

In July 2001, Yipsi Moreno of Cuba won her first major title in the hammer throw at the World Athletic Championships in Edmonton, Alberta, Canada, with a throw of 70.65 meters. The hammer is a metal ball, usually weighing 16 pounds, attached to a steel wire at the end of which is a grip. The ball is spun around by the thrower and then released, with the greatest distance thrown winning the event.



**TANGENTS** The figure models the hammer throw event. Circle  $A$  represents the circular area containing the spinning thrower. Ray  $BC$  represents the path the hammer takes when released.  $\overline{BC}$  is **tangent** to  $\odot A$ , because the line containing  $\overline{BC}$  intersects the circle in exactly one point. This point is called the **point of tangency**.



## Geometry Software Investigation

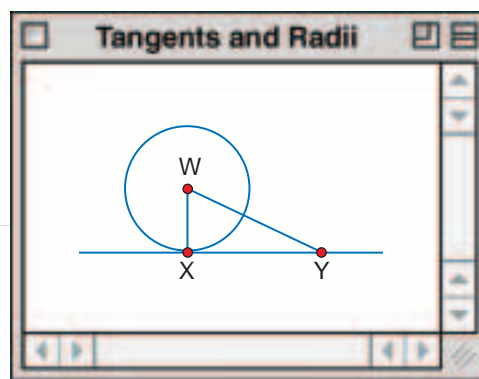
### Tangents and Radii

#### Model

- Use The Geometer's Sketchpad to draw a circle with center  $W$ . Then draw a segment tangent to  $\odot W$ . Label the point of tangency as  $X$ .
- Choose another point on the tangent and name it  $Y$ . Draw  $\overline{WY}$ .

#### Think and Discuss

1. What is  $\overline{WX}$  in relation to the circle?
2. Measure  $\overline{WY}$  and  $\overline{WX}$ . Write a statement to relate  $WX$  and  $WY$ .
3. Move point  $Y$  along the tangent. How does the location of  $Y$  affect the statement you wrote in Exercise 2?
4. Measure  $\angle WXY$ . What conclusion can you make?
5. **Make a conjecture** about the shortest distance from the center of the circle to a tangent of the circle.



## Study Tip

### Tangent Lines

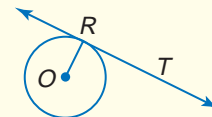
All of the theorems applying to tangent lines also apply to parts of the line that are tangent to the circle.

This investigation suggests an indirect proof of Theorem 10.9.

## Theorem 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

**Example:** If  $\overline{RT}$  is a tangent,  $\overline{OR} \perp \overline{RT}$ .



### Example 1 Find Lengths

**ALGEBRA**  $\overline{ED}$  is tangent to  $\odot F$  at point  $E$ . Find  $x$ .

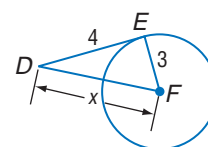
Because the radius is perpendicular to the tangent at the point of tangency,  $\overline{EF} \perp \overline{DE}$ . This makes  $\angle DEF$  a right angle and  $\triangle DEF$  a right triangle. Use the Pythagorean Theorem to find  $x$ .

$$(\overline{EF})^2 + (\overline{DE})^2 = (\overline{DF})^2 \quad \text{Pythagorean Theorem}$$

$$3^2 + 4^2 = x^2 \quad \overline{EF} = 3, \overline{DE} = 4, \overline{DF} = x$$

$$25 = x^2 \quad \text{Simplify.}$$

$$\pm 5 = x \quad \text{Take the square root of each side.}$$



Because  $x$  is the length of  $\overline{DF}$ , ignore the negative result. Thus,  $x = 5$ .

The converse of Theorem 10.9 is also true.

## Theorem 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

**Example:** If  $\overline{OR} \perp \overline{RT}$ ,  $\overline{RT}$  is a tangent.



You will prove this theorem in Exercise 22.

### Study Tip

#### Identifying Tangents

Never assume that a segment is tangent to a circle by appearance unless told otherwise. The figure must either have a right angle symbol or include the measurements that confirm a right angle.

### Example 2 Identify Tangents

a. Determine whether  $\overline{MN}$  is tangent to  $\odot L$ .

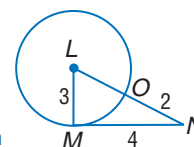
First determine whether  $\triangle LMN$  is a right triangle by using the converse of the Pythagorean Theorem.

$$(\overline{LM})^2 + (\overline{MN})^2 \stackrel{?}{=} (\overline{LN})^2 \quad \text{Converse of Pythagorean Theorem}$$

$$3^2 + 4^2 \stackrel{?}{=} 5^2 \quad \overline{LM} = 3, \overline{MN} = 4, \overline{LN} = 3 + 2 \text{ or } 5$$

$$25 = 25 \checkmark \quad \text{Simplify.}$$

Because the converse of the Pythagorean Theorem is true,  $\triangle LMN$  is a right triangle and  $\angle LMN$  is a right angle. Thus,  $\overline{LM} \perp \overline{MN}$ , making  $\overline{MN}$  a tangent to  $\odot L$ .



b. Determine whether  $\overline{PQ}$  is tangent to  $\odot R$ .

Since  $RQ = RS$ ,  $RP = 4 + 4$  or 8 units.

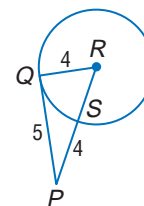
$$(\overline{RQ})^2 + (\overline{PQ})^2 \stackrel{?}{=} (\overline{RP})^2 \quad \text{Converse of Pythagorean Theorem}$$

$$4^2 + 5^2 \stackrel{?}{=} 8^2 \quad \overline{RQ} = 4, \overline{PQ} = 5, \overline{RP} = 8$$

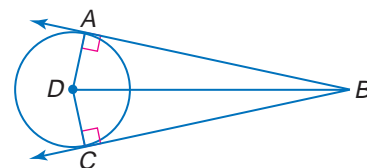
$$41 \neq 64 \quad \text{Simplify.}$$

Because the converse of the Pythagorean Theorem did not prove true in this case,  $\triangle RQP$  is not a right triangle.

So,  $\overline{PQ}$  is not tangent to  $\odot R$ .



More than one line can be tangent to the same circle. In the figure,  $\overline{AB}$  and  $\overline{BC}$  are tangent to  $\odot D$ . So,  $(AB)^2 + (AD)^2 = (DB)^2$  and  $(BC)^2 + (CD)^2 = (DB)^2$ .



$$(AB)^2 + (AD)^2 = (BC)^2 + (CD)^2 \quad \text{Substitution}$$

$$(AB)^2 + (AD)^2 = (BC)^2 + (AD)^2 \quad AD = CD$$

$$(AB)^2 = (BC)^2 \quad \text{Subtract } (AD)^2 \text{ from each side.}$$

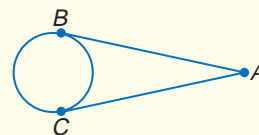
$$AB = BC \quad \text{Take the square root of each side.}$$

The last statement implies that  $\overline{AB} \cong \overline{BC}$ . This is a proof of Theorem 10.10.

### Theorem 10.11

If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example:  $\overline{AB} \cong \overline{AC}$



You will prove this theorem in Exercise 27.

### Example 3 Solve a Problem Involving Tangents

**ALGEBRA** Find  $x$ . Assume that segments that appear tangent to circles are tangent.

$\overline{AD}$  and  $\overline{AC}$  are drawn from the same exterior point and are tangent to  $\odot Q$ , so  $\overline{AD} \cong \overline{AC}$ .  $\overline{AC}$  and  $\overline{AB}$  are drawn from the same exterior point and are tangent to  $\odot R$ , so  $\overline{AC} \cong \overline{AB}$ . By the Transitive Property,  $\overline{AD} \cong \overline{AB}$ .

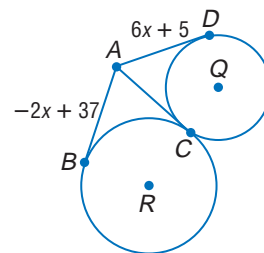
$$AD = AB \quad \text{Definition of congruent segments}$$

$$6x + 5 = -2x + 37 \quad \text{Substitution}$$

$$8x + 5 = 37 \quad \text{Add } 2x \text{ to each side.}$$

$$8x = 32 \quad \text{Subtract 5 from each side.}$$

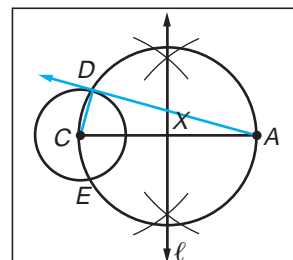
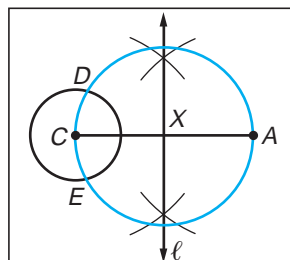
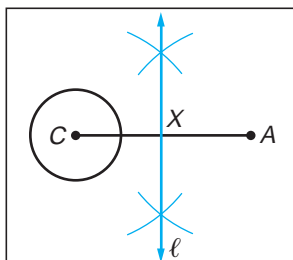
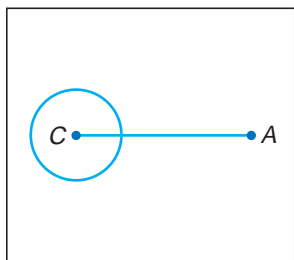
$$x = 4 \quad \text{Divide each side by 8.}$$



## Construction

### Line Tangent to a Circle Through a Point Exterior to the Circle

- Construct a circle. Label the center  $C$ . Draw a point outside  $\odot C$ . Then draw  $\overline{CA}$ .
- Construct the perpendicular bisector of  $\overline{CA}$  and label it line  $\ell$ . Label the intersection of  $\ell$  and  $\overline{CA}$  as point  $X$ .
- Construct circle  $X$  with radius  $XC$ . Label the points where the circles intersect as  $D$  and  $E$ .
- Draw  $\overline{AD}$ .  $\triangle ADC$  is inscribed in a semicircle. So  $\angle ADC$  is a right angle, and  $\overline{AD}$  is a tangent.



You will construct a line tangent to a circle through a point on the circle in Exercise 21.

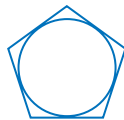
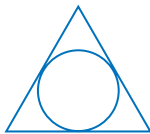


## Study Tip

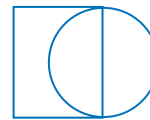
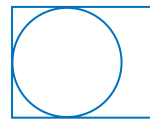
### Common Misconceptions

Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second pair of figures.

**CIRCUMSCRIBED POLYGONS** In Lesson 10-3, you learned that circles can be circumscribed about a polygon. Likewise, polygons can be circumscribed about a circle, or the circle is inscribed in the polygon. Notice that the vertices of the polygon do *not* lie on the circle, but every side of the polygon is tangent to the circle.



Polygons are circumscribed.



Polygons are *not* circumscribed.

### Example 4 Triangles Circumscribed About a Circle

Triangle  $ADC$  is circumscribed about  $\odot O$ . Find the perimeter of  $\triangle ADC$  if  $EC = DE + AF$ .

Use Theorem 10.10 to determine the equal measures.

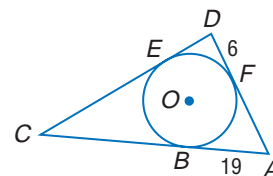
$AB = AF = 19$ ,  $FD = DE = 6$ , and  $EC = CB$ .

We are given that  $EC = DE + AF$ , so  $EC = 6 + 19$  or  $25$ .

$P = AB + BC + EC + DE + FD + AF$  Definition of perimeter

$= 19 + 25 + 25 + 6 + 6 + 19$  or  $100$  Substitution

The perimeter of  $\triangle ADC$  is 100 units.

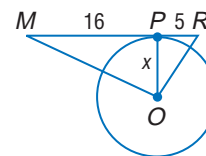


## Check for Understanding

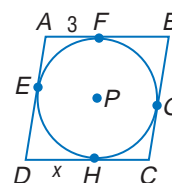
- Concept Check**
- Determine the number of tangents that can be drawn to a circle for each point. Explain your reasoning.
    - containing a point outside the circle
    - containing a point inside the circle
    - containing a point on the circle
  - Write an argument to support or provide a counterexample to the statement *If two lines are tangent to the same circle, they intersect.*
  - OPEN ENDED** Draw an example of a circumscribed polygon and an example of an inscribed polygon.

**Guided Practice** For Exercises 4 and 5, use the figure at the right.

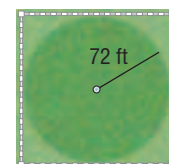
- Tangent  $\overline{MP}$  is drawn to  $\odot O$ . Find  $x$  if  $MO = 20$ .
- If  $RO = 13$ , determine whether  $\overline{PR}$  is tangent to  $\odot O$ .



- Rhombus  $ABCD$  is circumscribed about  $\odot P$  and has a perimeter of 32. Find  $x$ .



- Application**
- AGRICULTURE** A pivot-circle irrigation system waters part of a fenced square field. If the spray extends to a distance of 72 feet, what is the total length of the fence around the field?



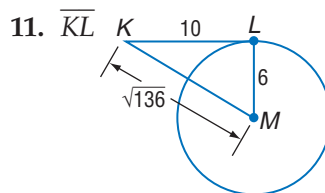
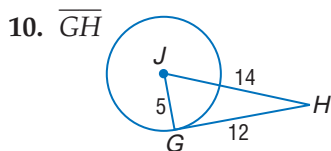
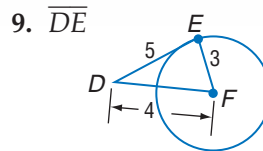
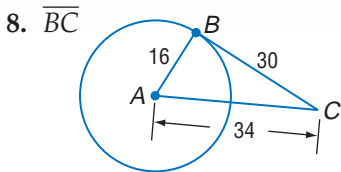
# Practice and Apply

## Homework Help

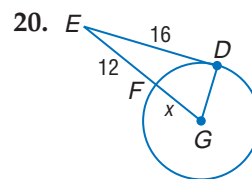
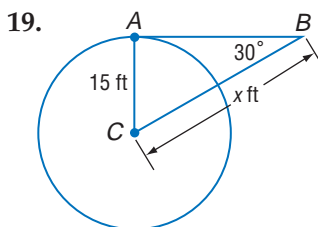
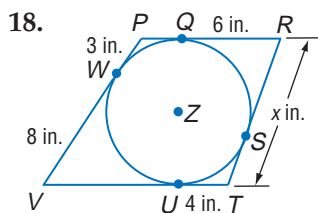
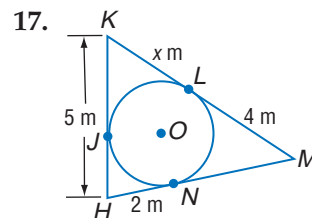
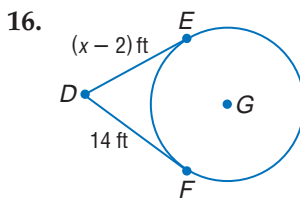
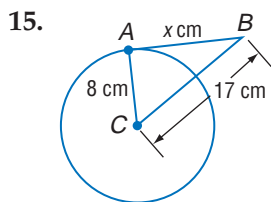
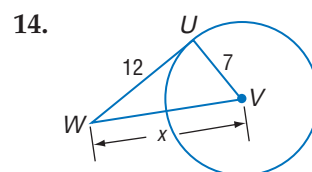
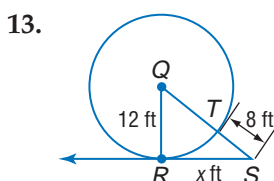
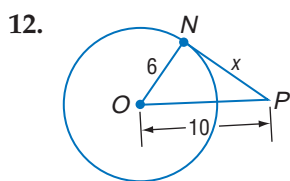
For Exercises	See Examples
8–11	2
12–20	1, 3
17, 18, 23–26	4

**Extra Practice**  
See page 775.

Determine whether each segment is tangent to the given circle.



Find  $x$ . Assume that segments that appear to be tangent are tangent.



## Study Tip

### Look Back

To review constructing perpendiculars to a line, see Lesson 3-6.



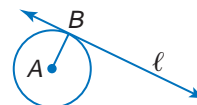
21. **CONSTRUCTION** Construct a line tangent to a circle through a point on the circle following these steps.

- Construct a circle with center  $T$ .
- Locate a point  $P$  on  $\odot T$  and draw  $\overline{TP}$ .
- Construct a perpendicular to  $\overline{TP}$  through point  $P$ .

22. **PROOF** Write an indirect proof of Theorem 10.10 by assuming that  $\ell$  is not tangent to  $\odot A$ .

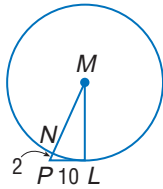
**Given:**  $\ell \perp \overline{AB}$ ,  $\overline{AB}$  is a radius of  $\odot A$ .

**Prove:** Line  $\ell$  is tangent to  $\odot A$ .

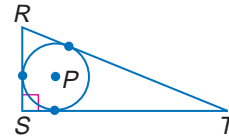


Find the perimeter of each polygon for the given information.

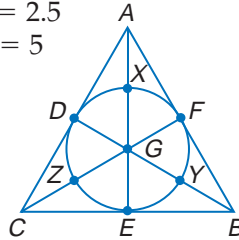
23.



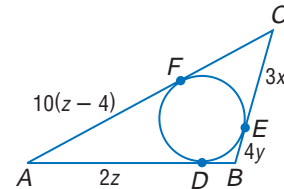
24.  $ST = 18$ , radius of  $\odot P = 5$



25.  $BY = CZ = AX = 2.5$   
diameter of  $\odot G = 5$

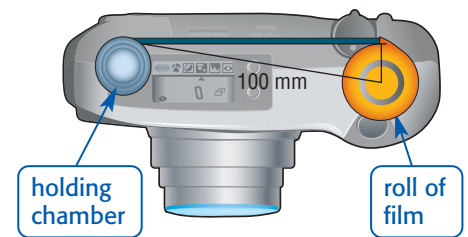


26.  $CF = 6(3 - x)$ ,  $DB = 12y - 4$



27. **PROOF** Write a two-column proof to show that if two segments from the same exterior point are tangent to a circle, then they are congruent. (Theorem 10.11)

28. **PHOTOGRAPHY** The film in a 35-mm camera unrolls from a cylinder, travels across an opening for exposure, and then is forwarded into another circular chamber as each photograph is taken. The roll of film has a diameter of 25 millimeters, and the distance from the center of the roll to the intake of the chamber is 100 millimeters. To the nearest millimeter, how much of the film would be exposed if the camera were opened before the roll had been totally used?



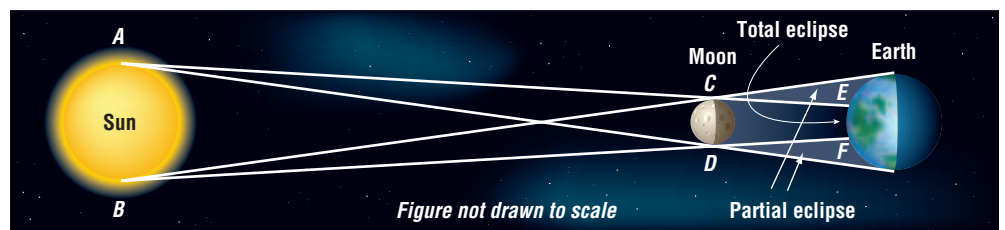
More About . . .

**Astronomy**

During the 20th century, there were 78 total solar eclipses, but only 15 of these affected parts of the United States. The next total solar eclipse visible in the U.S. will be in 2017.

Source: *World Almanac*

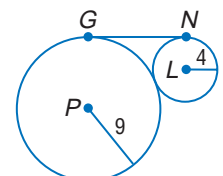
• **ASTRONOMY** For Exercises 29 and 30, use the following information. A solar eclipse occurs when the moon blocks the sun's rays from hitting Earth. Some areas of the world will experience a total eclipse, others a partial eclipse, and some no eclipse at all, as shown in the diagram below.



29. The blue section denotes a total eclipse on that part of Earth. Which tangents define the blue area?

30. The pink areas denote the portion of Earth that will have a partial eclipse. Which tangents define the northern and southern boundaries of the partial eclipse?

31. **CRITICAL THINKING** Find the measure of tangent  $\overline{GN}$ . Explain your reasoning.

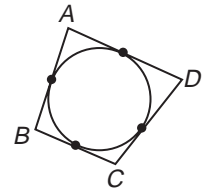


32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are tangents related to track and field events?

Include the following in your answer:

- how the hammer throw models a tangent, and
- the distance the hammer landed from the athlete if the wire and handle are 1.2 meters long and the athlete's arm is 0.8 meter long.



33. **GRID IN**  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{AD}$  are tangent to a circle. If  $AB = 19$ ,  $BC = 6$ , and  $CD = 14$ , find  $AD$ .
34. **ALGEBRA** Find the mean of all of the numbers from 1 to 1000 that end in 2.

- (A) 496                      (B) 497                      (C) 498                      (D) 500

**Extending the Lesson**

A line that is tangent to two circles in the same plane is called a *common tangent*.

Common internal tangents intersect the segment connecting the centers.	Common external tangents do not intersect the segment connecting the centers.
<p>Lines <math>k</math> and <math>j</math> are common internal tangents.</p>	<p>Lines <math>l</math> and <math>m</math> are common external tangents.</p>

Refer to the diagram of the eclipse on page 557.

35. Name two common internal tangents. 36. Name two common external tangents.

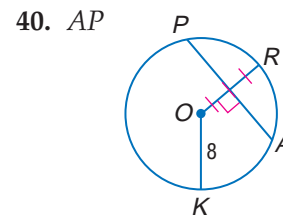
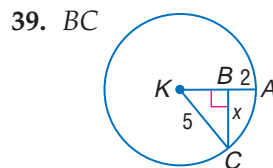
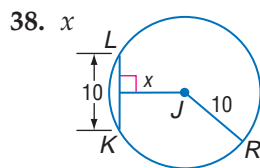
**Maintain Your Skills**

**Mixed Review**

37. **LOGOS** Circles are often used in logos for commercial products. The logo at the right shows two inscribed angles and two central angles. If  $\widehat{AC} \cong \widehat{BD}$ ,  $m\widehat{AF} = 90$ ,  $m\widehat{FE} = 45$ , and  $m\widehat{ED} = 90$ , find  $m\angle AFC$  and  $m\angle BED$ . (Lesson 10-4)



Find each measure. (Lesson 10-3)



41. **PROOF** Write a coordinate proof to show that if  $E$  is the midpoint of  $\overline{AB}$  in rectangle  $ABCD$ , then  $\triangle CED$  is isosceles. (Lesson 8-7)

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation.

(To review *solving equations*, see pages 737 and 738.)

42.  $x + 3 = \frac{1}{2}[(4x + 6) - 10]$                       43.  $2x - 5 = \frac{1}{2}[(3x + 16) - 20]$
44.  $2x + 4 = \frac{1}{2}[(x + 20) - 10]$                       45.  $x + 3 = \frac{1}{2}[(4x + 10) - 45]$



# Geometry Activity

A Follow-Up of Lessons 10-4 and 10-5

## Inscribed and Circumscribed Triangles

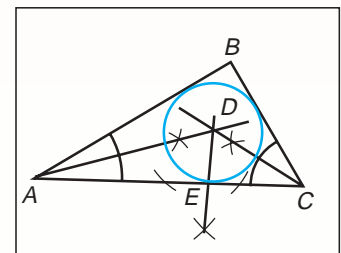
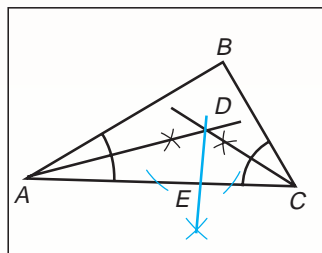
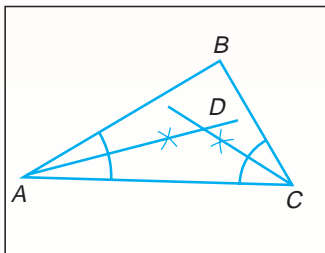
In Lesson 5-1, you learned that there are special points of concurrency in a triangle. Two of these will be used in these activities.

- The *incenter* is the point at which the angle bisectors meet. It is equidistant from the sides of the triangle.
- The *circumcenter* is the point at which the perpendicular bisectors of the sides intersect. It is equidistant from the vertices of the triangle.

### Activity 1

Construct a circle inscribed in a triangle. *The triangle is circumscribed about the circle.*

- 1 Draw a triangle and label its vertices  $A$ ,  $B$ , and  $C$ . Construct two angle bisectors of the triangle to locate the incenter. Label it  $D$ .
- 2 Construct a segment perpendicular to a side of  $\triangle ABC$  through the incenter. Label the intersection  $E$ .
- 3 Use the compass to measure  $DE$ . Then put the point of the compass on  $D$ , and draw a circle with that radius.

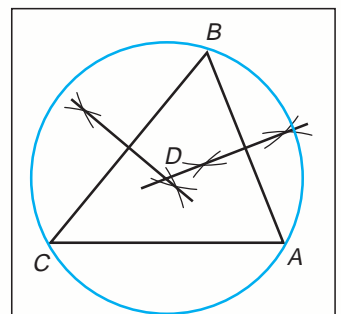
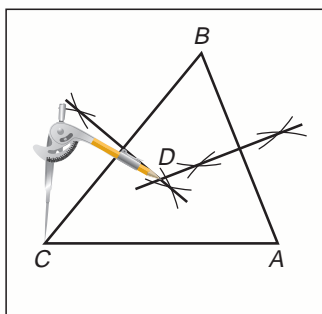
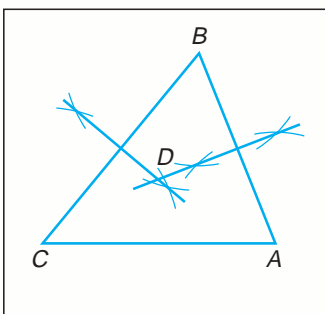


### Activity 2

Construct a circle through any three noncollinear points.

*This construction may be referred to as circumscribing a circle about a triangle.*

- 1 Draw a triangle and label its vertices  $A$ ,  $B$ , and  $C$ . Construct perpendicular bisectors of two sides of the triangle to locate the circumcenter. Label it  $D$ .
- 2 Use the compass to measure the distance from the circumcenter  $D$  to any of the three vertices.
- 3 Using that setting, place the compass point at  $D$ , and draw a circle about the triangle.



*(continued on the next page)*



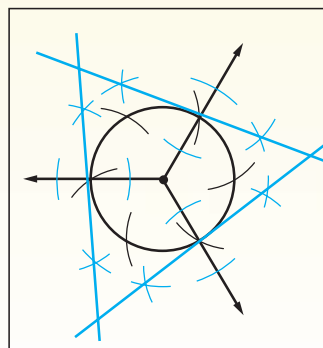
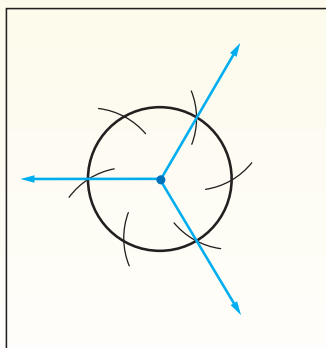
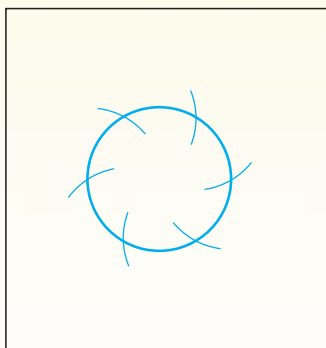
## Geometry Activity

For the next activity, refer to the construction of an inscribed regular hexagon on page 542.

### Activity 3

Construct an equilateral triangle circumscribed about a circle.

- 1 Construct a circle and divide it into six congruent arcs.
- 2 Place a point at every other arc. Draw rays from the center through these points.
- 3 Construct a line perpendicular to each of the rays through the points.



### Model

1. Draw an obtuse triangle and inscribe a circle in it.
2. Draw a right triangle and circumscribe a circle about it.
3. Draw a circle of any size and circumscribe an equilateral triangle about it.

### Analyze

Refer to Activity 1.

4. Why do you only have to construct the perpendicular to one side of the triangle?
5. How can you use the Incenter Theorem to explain why this construction is valid?

Refer to Activity 2.

6. Why do you only have to measure the distance from the circumcenter to any one vertex?
7. How can you use the Circumcenter Theorem to explain why this construction is valid?

Refer to Activity 3.

8. What is the measure of each of six congruent arcs?
9. Write a convincing argument as to why the lines constructed in Step 3 form an equilateral triangle.
10. Why do you think the terms *incenter* and *circumcenter* are good choices for the points they define?

# Secants, Tangents, and Angle Measures

## What You'll Learn

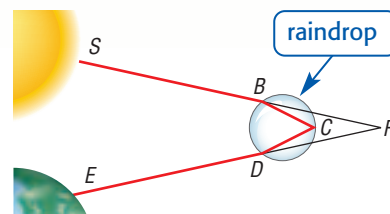
- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

## Vocabulary

- secant

## How is a rainbow formed by segments of a circle?

Droplets of water in the air refract or bend sunlight as it passes through them, creating a rainbow. The various angles of refraction result in an arch of colors. In the figure, the sunlight from point  $S$  enters the raindrop at  $B$  and is bent. The light proceeds to the back of the raindrop, and is reflected at  $C$  to leave the raindrop at point  $D$  heading to Earth. Angle  $F$  represents the measure of how the resulting ray of light deviates from its original path.

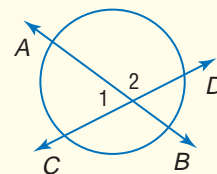


**INTERSECTIONS ON OR INSIDE A CIRCLE** A line that intersects a circle in exactly two points is called a **secant**. In the figure above,  $\overline{SF}$  and  $\overline{EF}$  are secants of the circle. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

## Theorem 10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

**Examples:**  $m\angle 1 = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$   
 $m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$



## Proof Theorem 10.12

**Given:** secants  $\overline{RT}$  and  $\overline{SU}$

**Prove:**  $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$

Draw  $\overline{RS}$ . Label  $\angle TRS$  as  $\angle 2$  and  $\angle USR$  as  $\angle 3$ .

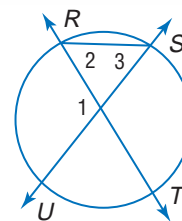
**Proof:**

### Statements

- $m\angle 1 = m\angle 2 + m\angle 3$
- $m\angle 2 = \frac{1}{2}m\widehat{ST}$ ,  $m\angle 3 = \frac{1}{2}m\widehat{RU}$
- $m\angle 1 = \frac{1}{2}m\widehat{ST} + \frac{1}{2}m\widehat{RU}$
- $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$

### Reasons

- Exterior Angle Theorem
- The measure of inscribed  $\angle$  = half the measure of the intercepted arc.
- Substitution
- Distributive Property



### Example 1 Secant-Secant Angle

Find  $m\angle 2$  if  $m\widehat{BC} = 30$  and  $m\widehat{AD} = 20$ .

**Method 1**

$$\begin{aligned} m\angle 1 &= \frac{1}{2}(m\widehat{BC} + m\widehat{AD}) \\ &= \frac{1}{2}(30 + 20) \text{ or } 25 \quad \text{Substitution} \end{aligned}$$

$$\begin{aligned} m\angle 2 &= 180 - m\angle 1 \\ &= 180 - 25 \text{ or } 155 \end{aligned}$$

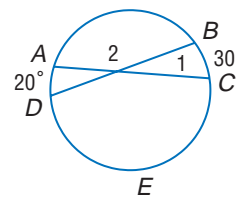
**Method 2**

$$m\angle 2 = \frac{1}{2}(m\widehat{AB} + m\widehat{DEC})$$

Find  $m\widehat{AB} + m\widehat{DEC}$ .

$$\begin{aligned} m\widehat{AB} + m\widehat{DEC} &= 360 - (m\widehat{BC} + m\widehat{AD}) \\ &= 360 - (30 + 20) \\ &= 360 - 50 \text{ or } 310 \end{aligned}$$

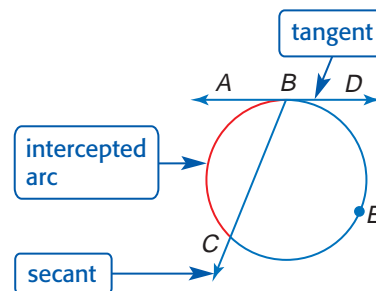
$$\begin{aligned} m\angle 2 &= \frac{1}{2}(m\widehat{AB} + m\widehat{DEC}) \\ &= \frac{1}{2}(310) \text{ or } 155 \end{aligned}$$



A secant can also intersect a tangent at the point of tangency. Angle  $ABC$  intercepts  $\widehat{BC}$ , and  $\angle DBC$  intercepts  $\widehat{BEC}$ . Each angle formed has a measure half that of the arc it intercepts.

$$m\angle ABC = \frac{1}{2}m\widehat{BC} \quad m\angle DBC = \frac{1}{2}m\widehat{BEC}$$

This is stated formally in Theorem 10.13.



### Theorem 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

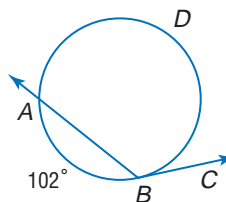
You will prove this theorem in Exercise 43.

### Example 2 Secant-Tangent Angle

Find  $m\angle ABC$  if  $m\widehat{AB} = 102$ .

$$\begin{aligned} m\widehat{ADB} &= 360 - m\widehat{AB} \\ &= 360 - 102 \text{ or } 258 \end{aligned}$$

$$\begin{aligned} m\angle ABC &= \frac{1}{2}m\widehat{ADC} \\ &= \frac{1}{2}(258) \text{ or } 129 \end{aligned}$$



**INTERSECTIONS OUTSIDE A CIRCLE** Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

### Study Tip

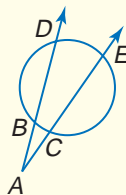
#### Absolute Value

The measure of each  $\angle A$  can also be expressed as one-half the absolute value of the difference of the arc measures. In this way, the order of the arc measures does not affect the outcome of the calculation.

### Theorem 10.14

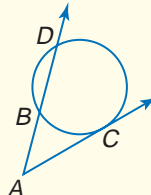
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

#### Two Secants



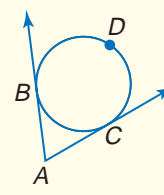
$$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$$

#### Secant-Tangent



$$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$$

#### Two Tangents



$$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$$

You will prove this theorem in Exercise 40.

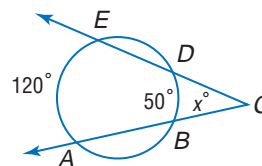
### Example 3 Secant-Secant Angle

Find  $x$ .

$$m\angle C = \frac{1}{2}(m\widehat{EA} - m\widehat{DB})$$

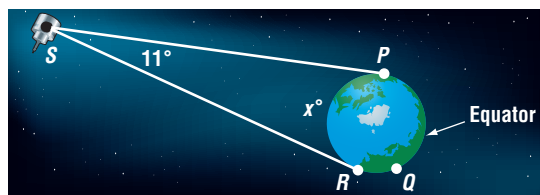
$$x = \frac{1}{2}(120 - 50) \quad \text{Substitution}$$

$$x = \frac{1}{2}(70) \text{ or } 35 \quad \text{Simplify.}$$



### Example 4 Tangent-Tangent Angle

**SATELLITES** Suppose a geostationary satellite  $S$  orbits about 35,000 kilometers above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this geostationary satellite.



$\widehat{PR}$  represents the arc along the equator visible to the satellite  $S$ . If  $x = m\widehat{PR}$ , then  $m\widehat{PQR} = 360 - x$ . Use the measure of the given angle to find  $m\widehat{PR}$ .

$$m\angle S = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$$

$$11 = \frac{1}{2}[(360 - x) - x] \quad \text{Substitution}$$

$$22 = 360 - 2x \quad \text{Multiply each side by 2 and simplify.}$$

$$-338 = -2x \quad \text{Subtract 360 from each side.}$$

$$169 = x \quad \text{Divide each side by } -2.$$

The measure of the arc on Earth visible to the satellite is 169.



### Example 5 Secant-Tangent Angle

Find  $x$ .

$\widehat{WRV}$  is a semicircle because  $\overline{WV}$  is a diameter.

So,  $m\widehat{WRV} = 180$ .

$$m\angle Y = \frac{1}{2}(m\widehat{WV} - m\widehat{ZV})$$

$$45 = \frac{1}{2}(180 - 10x)$$

Substitution

$$90 = 180 - 10x$$

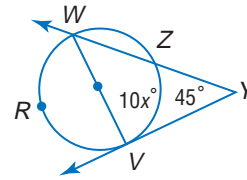
Multiply each side by 2.

$$-90 = -10x$$

Subtract 180 from each side.

$$9 = x$$

Divide each side by  $-10$ .



## Check for Understanding

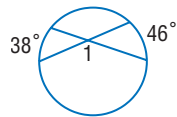
### Concept Check

- Describe the difference between a secant and a tangent.
- OPEN ENDED** Draw a circle and one of its diameters. Call the diameter  $\overline{AC}$ . Draw a line tangent to the circle at  $A$ . What type of angle is formed by the tangent and the diameter? Explain.

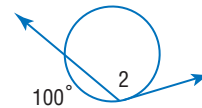
### Guided Practice

Find each measure.

3.  $m\angle 1$

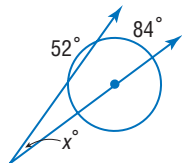


4.  $m\angle 2$

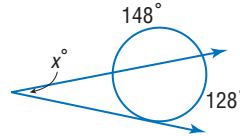


Find  $x$ .

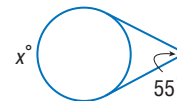
5.



6.



7.

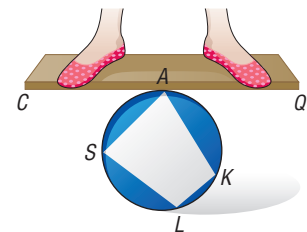


### Application

**CIRCUS** For Exercises 8–11, refer to the figure and the information below.

One of the acrobatic acts in the circus requires the artist to balance on a board that is placed on a round drum as shown at the right. Find each measure if  $\overline{SA} \parallel \overline{LK}$ ,  $m\angle SLK = 78$ , and  $m\widehat{SA} = 46$ .

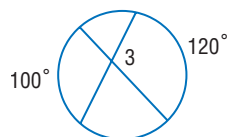
- $m\angle CAS$
- $m\angle QAK$
- $m\widehat{KL}$
- $m\widehat{SL}$



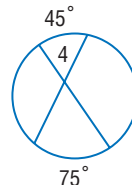
## Practice and Apply

Find each measure.

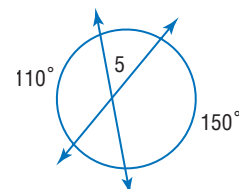
12.  $m\angle 3$



13.  $m\angle 4$



14.  $m\angle 5$



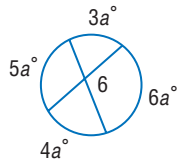


### Homework Help

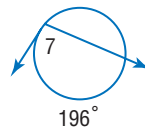
For Exercises	See Examples
12–15	1
16–19	2
21–24, 31	3
25–28, 32	5
29–30	4

**Extra Practice**  
See page 775.

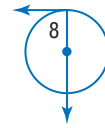
15.  $m\angle 6$



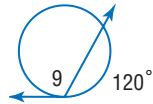
16.  $m\angle 7$



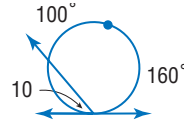
17.  $m\angle 8$



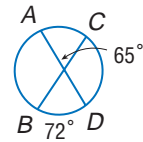
18.  $m\angle 9$



19.  $m\angle 10$

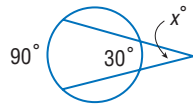


20.  $m\widehat{AC}$

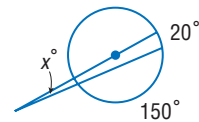


Find  $x$ . Assume that any segment that appears to be tangent is tangent.

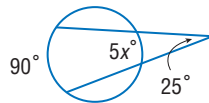
21.



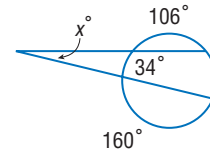
22.



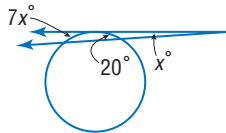
23.



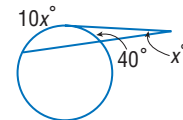
24.



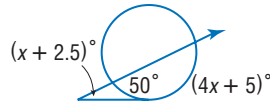
25.



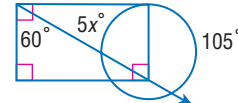
26.



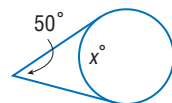
27.



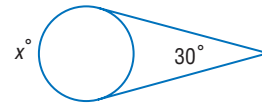
28.



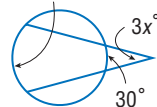
29.



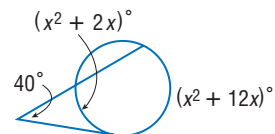
30.



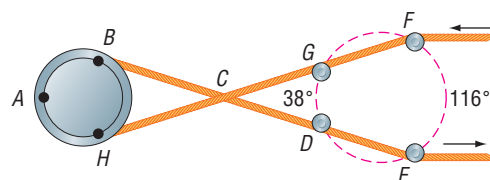
31.  $(4x + 50)^\circ$



32.



33. **WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown below. Note that the yarn appears to intersect at  $C$ , but in reality it does not. Use the information from the diagram to find  $m\widehat{BH}$ .



## More About . . .



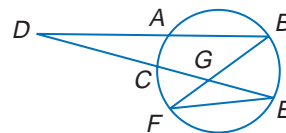
### Landmarks

Stonehenge is located in southern England near Salisbury. In its final form, Stonehenge included 30 upright stones about 18 feet tall by 7 feet thick.

Source: *World Book Encyclopedia*

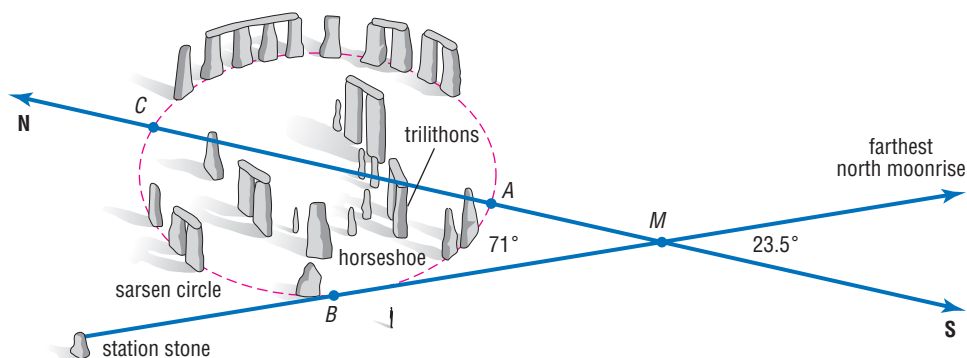
Find each measure if  $m\widehat{FE} = 118$ ,  $m\widehat{AB} = 108$ ,  $m\angle EGB = 52$ , and  $m\angle EFB = 30$ .

34.  $m\widehat{AC}$
35.  $m\widehat{CF}$
36.  $m\angle EDB$



• **LANDMARKS** For Exercises 37–39, use the following information.

Stonehenge is a British landmark made of huge stones arranged in a circular pattern that reflects the movements of Earth and the moon. The diagram shows that the angle formed by the north/south axis and the line aligned from the station stone to the northmost moonrise position measures  $23.5^\circ$ .

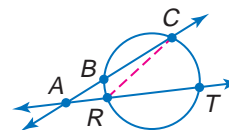


37. Find  $m\widehat{BC}$ .
38. Is  $\widehat{ABC}$  a semicircle? Explain.
39. If the circle measures about 100 feet across, approximately how far would you walk around the circle from point  $B$  to point  $C$ ?
40. **PROOF** Write a two-column proof of Theorem 10.14. Consider each case.

a. Case 1: Two Secants

**Given:**  $\overline{AC}$  and  $\overline{AT}$  are secants to the circle.

**Prove:**  $m\angle CAT = \frac{1}{2}(m\widehat{CT} - m\widehat{BR})$

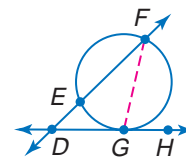


b. Case 2: Secant and a Tangent

**Given:**  $\overline{DG}$  is a tangent to the circle.

$\overline{DF}$  is a secant to the circle.

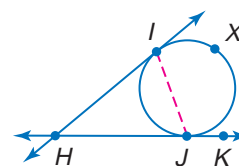
**Prove:**  $m\angle FDG = \frac{1}{2}(m\widehat{FG} - m\widehat{GE})$



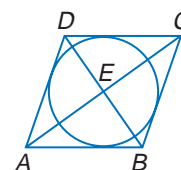
c. Case 3: Two Tangents

**Given:**  $\overline{HI}$  and  $\overline{HJ}$  are tangents to the circle.

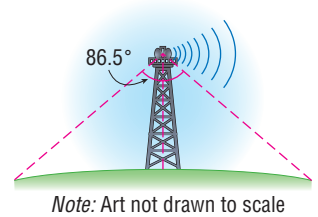
**Prove:**  $m\angle IHJ = \frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ})$



41. **CRITICAL THINKING** Circle  $E$  is inscribed in rhombus  $ABCD$ . The diagonals of the rhombus are 10 centimeters and 24 centimeters long. To the nearest tenth centimeter, how long is the radius of circle  $E$ ? (*Hint:* Draw an altitude from  $E$ .)



42. **TELECOMMUNICATION** The signal from a telecommunication tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level as shown in the figure. Determine the measure of the arc intercepted by the two tangents.

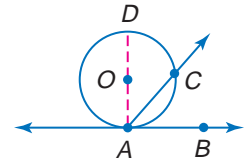


43. **PROOF** Write a paragraph proof of Theorem 10.13

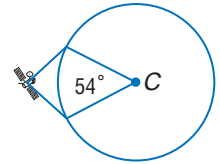
- a. **Given:**  $\overline{AB}$  is a tangent of  $\odot O$ .  
 $\overline{AC}$  is a secant of  $\odot O$ .  
 $\angle CAB$  is acute.

**Prove:**  $m\angle CAB = \frac{1}{2}m\widehat{CA}$

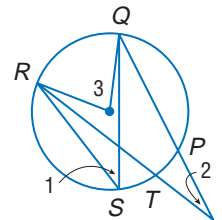
- b. Prove Theorem 10.13 if the angle in part a is obtuse.



44. **SATELLITES** A satellite is orbiting so that it maintains a constant altitude above the equator. The camera on the satellite can detect an arc of 6000 kilometers on Earth's surface. This arc measures  $54^\circ$ . What is the measure of the angle of view of the camera located on the satellite?



45. **CRITICAL THINKING** In the figure,  $\angle 3$  is a central angle. List the numbered angles in order from greatest measure to least measure. Explain your reasoning.



46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is a rainbow formed by segments of a circle?**

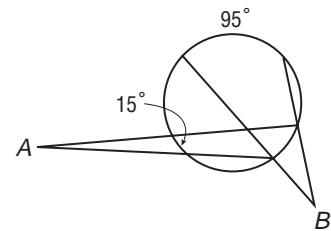
Include the following in your answer:

- the types of segments represented in the figure on page 561, and
- how you would calculate the angle representing how the light deviates from its original path.



47. What is the measure of  $\angle B$  if  $m\angle A = 10^\circ$ ?

- (A) 30                      (B) 35  
 (C) 47.5                    (D) 90



48. **ALGEBRA** Which of the following sets of data can be represented by a linear equation?

(A)

x	y
1	2
2	4
3	8
4	16

(B)

x	y
1	4
2	2
3	2
4	4

(C)

x	y
2	2
4	3
6	4
8	5

(D)

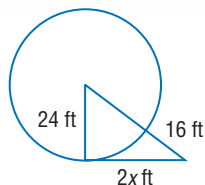
x	y
1	1
3	9
5	25
7	49



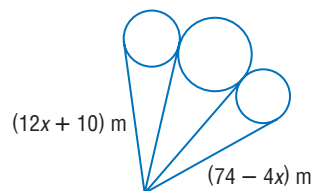
# Maintain Your Skills

**Mixed Review** Find  $x$ . Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

49.



50.

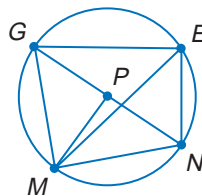


In  $\odot P$ ,  $m\widehat{EN} = 66$  and  $m\angle GPM = 89$ . Find each measure. (Lesson 10-4)

51.  $m\angle EGN$

52.  $m\angle GME$

53.  $m\angle GNM$



**RAMPS** Use the following information for Exercises 54 and 55.

The Americans with Disabilities Act (ADA), which went into effect in 1990, requires that ramps should have at least a 12-inch run for each rise of 1 inch. (Lesson 3-3)

54. Determine the slope represented by this requirement.

55. The maximum length the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp?

56. **PROOF** Write a paragraph proof to show that  $AB = CF$  if  $\overline{AC} \cong \overline{BF}$ . (Lesson 2-5)



**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Solve each equation by factoring.

(To review **solving equations by factoring**, see pages 750 and 751.)

57.  $x^2 + 6x - 40 = 0$

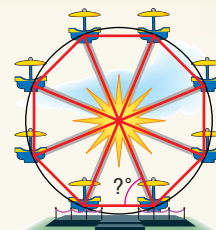
58.  $2x^2 + 7x - 30 = 0$

59.  $3x^2 - 24x + 45 = 0$

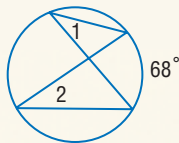
## Practice Quiz 2

Lessons 10-4 through 10-6

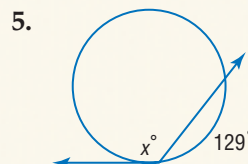
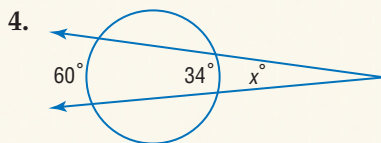
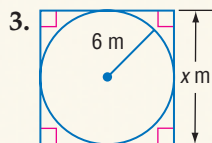
1. **AMUSEMENT RIDES** A Ferris wheel is shown at the right. If the distances between the seat axles are the same, what is the measure of an angle formed by the braces attaching consecutive seats? (Lesson 10-4)



2. Find the measure of each numbered angle. (Lesson 10-4)



Find  $x$ . Assume that any segment that appears to be tangent is tangent. (Lessons 10-5 and 10-6)

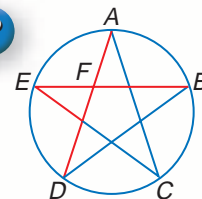


**What** You'll Learn

- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

**How** are lengths of intersecting chords related?

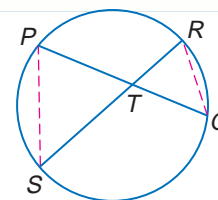
The star is inscribed in a circle. It was formed by intersecting chords. Segments  $AD$  and  $EB$  are two of those chords. When two chords intersect, four smaller segments are defined.



**SEGMENTS INTERSECTING INSIDE A CIRCLE** In Lesson 10-2, you learned how to find lengths of parts of a chord that is intersected by the perpendicular diameter. But how do you find lengths for other intersecting chords?

**Geometry Activity****Intersecting Chords****• Make A Model**

- Draw a circle and two intersecting chords.
- Name the chords  $\overline{PQ}$  and  $\overline{RS}$  intersecting at  $T$ .
- Draw  $\overline{PS}$  and  $\overline{RQ}$ .

**Analyze**

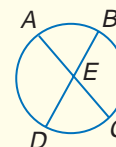
1. Name pairs of congruent angles. Explain your reasoning.
2. How are  $\triangle PTS$  and  $\triangle RTQ$  related? Why?
3. **Make a conjecture** about the relationship of  $\overline{PT}$ ,  $\overline{TQ}$ ,  $\overline{RT}$ , and  $\overline{ST}$ .

The results of the Geometry Activity suggest a proof for Theorem 10.15.

**Theorem 10.15**

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

**Example:**  $AE \cdot EC = BE \cdot ED$



You will prove Theorem 10.15 in Exercise 21.

**Example 1** Intersection of Two Chords

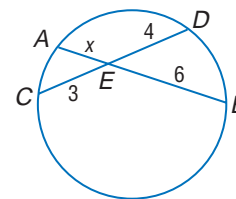
Find  $x$ .

$$AE \cdot EB = CE \cdot ED$$

$$x \cdot 6 = 3 \cdot 4 \quad \text{Substitution}$$

$$6x = 12 \quad \text{Multiply.}$$

$$x = 2 \quad \text{Divide each side by 6.}$$

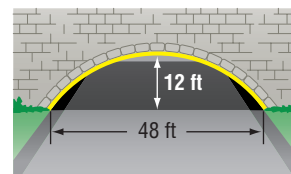




Intersecting chords can also be used to measure arcs.

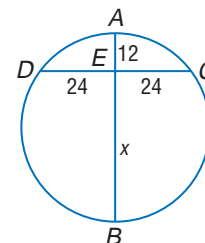
### Example 2 Solve Problems

**TUNNELS** Tunnels are constructed to allow roadways to pass through mountains. What is the radius of the circle containing the arc if the opening is not a semicircle?



Draw a model using a circle. Let  $x$  represent the unknown measure of the segment of diameter  $\overline{AB}$ . Use the products of the lengths of the intersecting chords to find the length of the diameter.

$$\begin{aligned} AE \cdot EB &= DE \cdot EC && \text{Segment products} \\ 12x &= 24 \cdot 24 && \text{Substitution} \\ x &= 48 && \text{Divide each side by 12.} \\ AB &= AE + EB && \text{Segment Addition Postulate} \\ AB &= 12 + 48 \text{ or } 60 && \text{Substitution and addition} \end{aligned}$$



Since the diameter is 60,  $r = 30$ .

**SEGMENTS INTERSECTING OUTSIDE A CIRCLE** Chords of a circle can be extended to form secants that intersect in the exterior of a circle. The special relationship among secant segments excludes the chord.

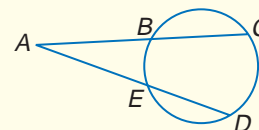
### Study Tip

#### Helping You Remember

To remember this concept, the wording of Theorem 19.16 can be simplified by saying that each side of the equation is the product of the exterior part and the whole segment.

### Theorem 10.16

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



**Example:**  $AB \cdot AC = AE \cdot AD$

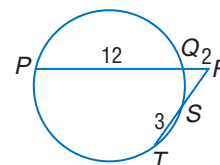
You will prove this theorem in Exercise 30.

### Example 3 Intersection of Two Secants

Find  $RS$  if  $PQ = 12$ ,  $QR = 2$ , and  $TS = 3$ .

Let  $RS = x$ .

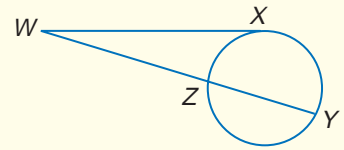
$$\begin{aligned} QR \cdot PR &= RS \cdot RT && \text{Secant Segment Products} \\ 2 \cdot (12 + 2) &= x \cdot (x + 3) && \text{Substitution} \\ 28 &= x^2 + 3x && \text{Distributive Property} \\ 0 &= x^2 + 3x - 28 && \text{Subtract 28 from each side.} \\ 0 &= (x + 7)(x - 4) && \text{Factor.} \\ x + 7 &= 0 && x - 4 = 0 \\ x &= -7 && x = 4 && \text{Disregard negative value.} \end{aligned}$$



The same secant segment product can be used with a secant segment and a tangent. In this case, the tangent is both the exterior part and the whole segment. This is stated in Theorem 10.17.

### Theorem 10.17

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.



**Example:**  $WX \cdot WX = WZ \cdot WY$

You will prove this theorem in Exercise 31.

### Example 4 Intersection of a Secant and a Tangent.

Find  $x$ . Assume that segments that appear to be tangent are tangent.

$$(AB)^2 = BC \cdot BD$$

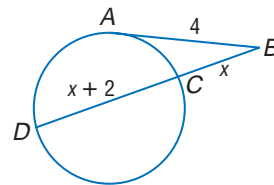
$$4^2 = x(x + x + 2)$$

$$16 = x(2x + 2)$$

$$16 = 2x^2 + 2x$$

$$0 = 2x^2 + 2x - 16$$

$$0 = x^2 + x - 8$$



This expression is not factorable. Use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)}$$

$$a = 1, b = 1, c = -8$$

$$= \frac{-1 + \sqrt{33}}{2} \text{ and } x = \frac{-1 - \sqrt{33}}{2}$$

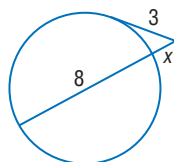
Disregard the negative solution.

$$\approx 2.37$$

Use a calculator.

## Check for Understanding

- Concept Check**
- Show how the products for secant segments are similar to the products for a tangent and a secant segment.
  - FIND THE ERROR** Becky and Latisha are writing products to find  $x$ . Who is correct? Explain your reasoning.



Becky

$$3^2 = x \cdot 8$$

$$9 = 8x$$

$$\frac{9}{8} = x$$

Latisha

$$3^2 = x(x + 8)$$

$$9 = x^2 + 8x$$

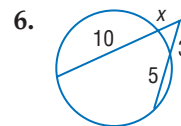
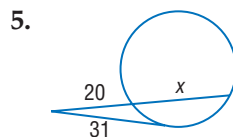
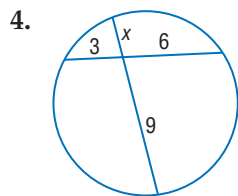
$$0 = x^2 + 8x - 9$$

$$0 = (x + 9)(x - 1)$$

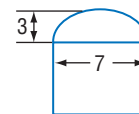
$$x = 1$$

3. **OPEN ENDED** Draw a circle with two secant segments and one tangent segment that intersect at the same point.

**Guided Practice** Find  $x$ . Round to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.



- Application** 7. **HISTORY** The Roman Coliseum has many “entrances” in the shape of a door with an arched top. The ratio of the arch width to the arch height is 7:3. Find the ratio of the arch width to the radius of the circle that contains the arch.



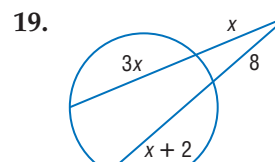
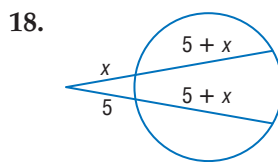
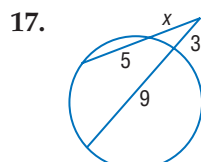
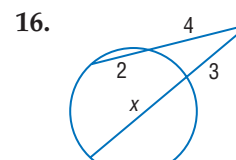
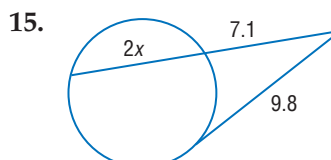
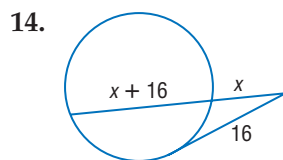
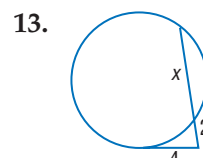
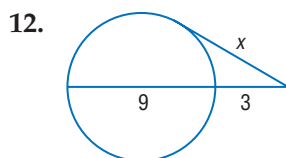
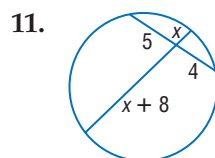
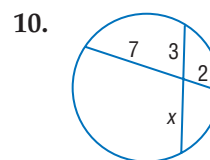
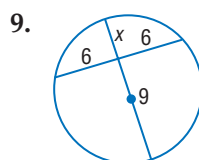
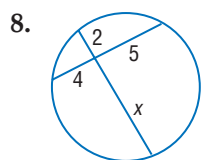
## Practice and Apply

### Homework Help

For Exercises	See Examples
8–11	1
12–15	4
16–19	3
20, 27	2

**Extra Practice**  
See page 775.

Find  $x$ . Round to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.



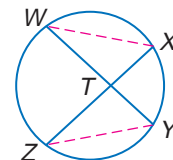
20. **KNOBS** If you remove a knob from a kitchen appliance, you may notice that the hole is not completely round. Suppose the flat edge is 4 millimeters long and the distance from the curved edge to the flat edge is about 4.25 millimeters. Find the radius of the circle containing the hole.



21. **PROOF** Copy and complete the proof of Theorem 10.15.

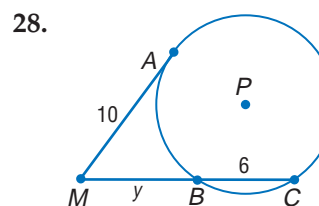
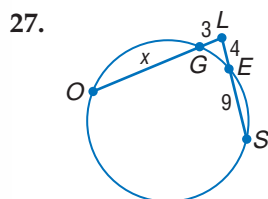
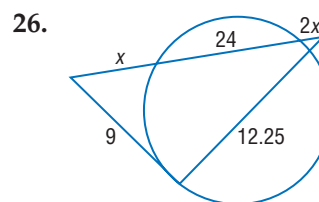
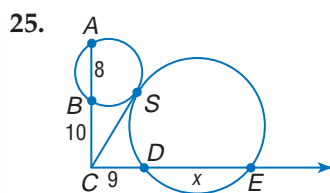
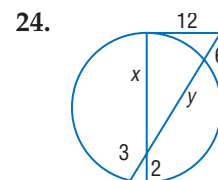
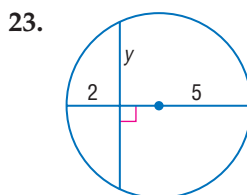
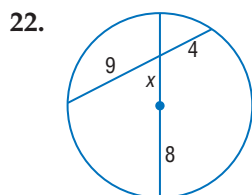
**Given:**  $\overline{WY}$  and  $\overline{ZX}$  intersect at  $T$ .

**Prove:**  $WT \cdot TY = ZT \cdot TX$

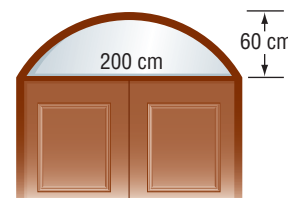


Statements	Reasons
a. $\angle W \cong \angle Z, \angle X \cong \angle Y$	a. <u>?</u>
b. <u>?</u>	b. AA Similarity
c. $\frac{WT}{ZT} = \frac{TX}{TY}$	c. <u>?</u>
d. <u>?</u>	d. Cross products

Find each variable to the nearest tenth.



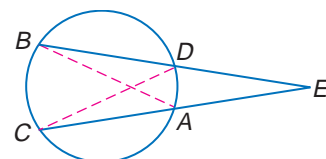
29. **CONSTRUCTION** An arch over a courtroom door is 60 centimeters high and 200 centimeters wide. Find the radius of the circle containing the arc of the arch.



30. **PROOF** Write a paragraph proof of Theorem 10.16.

**Given:** secants  $\overline{EC}$  and  $\overline{EB}$

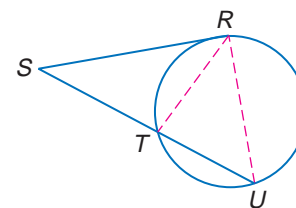
**Prove:**  $EA \cdot EC = ED \cdot EB$



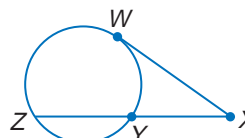
31. **PROOF** Write a two-column proof of Theorem 10.17.

**Given:** tangent  $\overline{RS}$ , secant  $\overline{SU}$

**Prove:**  $(RS)^2 = ST \cdot SU$



32. **CRITICAL THINKING** In the figure,  $Y$  is the midpoint of  $\overline{XZ}$ . Find  $WX$ . Explain your reasoning.



### Career Choices



### Construction Worker

Construction workers must know how to measure and fit shapes together to make a sound building that will last for years to come. These workers also must master using machines to cut wood and metal to certain specifications that are based on geometry.

### Online Research

For more information about a career as a construction worker, visit: [www.geometryonline.com/careers](http://www.geometryonline.com/careers)

33. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are the lengths of intersecting chords related?**

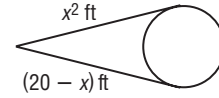
Include the following in your answer:

- the segments formed by intersecting segments,  $\overline{AD}$  and  $\overline{EB}$ , and
- the relationship among these segments.



34. Find two possible values for  $x$  from the information in the figure.

- (A)  $-4, -5$       (B)  $-4, 5$   
 (C)  $4, 5$       (D)  $4, -5$



35. **ALGEBRA** Mr. Rodriguez can wash his car in 15 minutes, while his son Marcus takes twice as long to do the same job. If they work together, how long will it take them to wash the car?

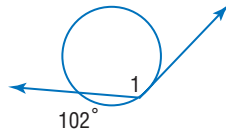
- (A) 5 min      (B) 7.5 min      (C) 10 min      (D) 22.5 min

## Maintain Your Skills

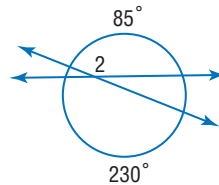
### Mixed Review

Find the measure of each numbered angle. Assume that segments that appear tangent are tangent. (Lesson 10-6)

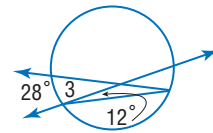
36.



37.

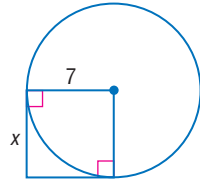


38.

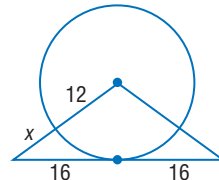


Find  $x$ . Assume that segments that appear to be tangent are tangent. (Lesson 10-5)

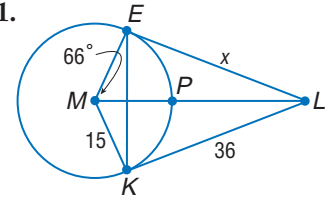
39.



40.



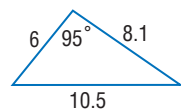
41.



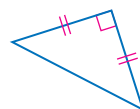
42. **INDIRECT MEASUREMENT** Joseph Blackarrow is measuring the width of a stream on his land to build a bridge over it. He picks out a rock across the stream as landmark  $A$  and places a stone on his side as point  $B$ . Then he measures 5 feet at a right angle from  $\overline{AB}$  and marks this  $C$ . From  $C$ , he sights a line to point  $A$  on the other side of the stream and measures the angle to be about  $67^\circ$ . How far is it across the stream rounded to the nearest whole foot? (Lesson 7-5)

Classify each triangle by its sides and by its angles. (Lesson 4-1)

43.



44.



45.



### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Find the distance between each pair of points.

(To review the **Distance Formula**, see Lesson 1-3.)

46.  $C(-2, 7), D(10, 12)$       47.  $E(1, 7), F(3, 4)$       48.  $G(9, -4), H(15, -2)$



# 10-8 Equations of Circles

## What You'll Learn

- Write the equation of a circle.
- Graph a circle on the coordinate plane.

## What kind of equations describes the ripples of a splash?

When a rock enters the water, ripples move out from the center forming concentric circles. If the rock is assigned coordinates, each ripple can be modeled by an equation of a circle.



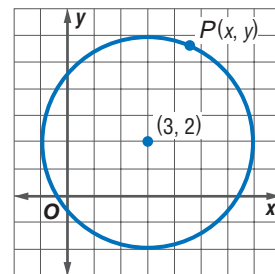
**EQUATION OF A CIRCLE** The fact that a circle is the *locus* of points in a plane equidistant from a given point creates an equation for any circle.

Suppose the center is at  $(3, 2)$  and the radius is 4. The radius is the distance from the center. Let  $P(x, y)$  be the endpoint of any radius.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$4 = \sqrt{(x - 3)^2 + (y - 2)^2} \quad d = 4, (x_1, y_1) = (3, 2)$$

$$16 = (x - 3)^2 + (y - 2)^2 \quad \text{Square each side.}$$

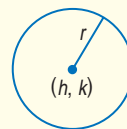


Applying this same procedure to an unknown center  $(h, k)$  and radius  $r$  yields a general equation for any circle.

## Key Concept

## Standard Equation of a Circle

An equation for a circle with center at  $(h, k)$  and radius of  $r$  units is  $(x - h)^2 + (y - k)^2 = r^2$ .



## Study Tip

### Equation of Circles

Note that the equation of a circle is kept in the form shown above. The terms being squared are not expanded.

## Example 1 Equation of a Circle

Write an equation for each circle.

- a. center at  $(-2, 4)$ ,  $d = 4$

$$\text{If } d = 4, r = 2.$$

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$[x - (-2)]^2 + [y - 4]^2 = 2^2 \quad (h, k) = (-2, 4), r = 2$$

$$(x + 2)^2 + (y - 4)^2 = 4 \quad \text{Simplify.}$$

- b. center at origin,  $r = 3$

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x - 0)^2 + (y - 0)^2 = 3^2 \quad (h, k) = (0, 0), r = 3$$

$$x^2 + y^2 = 9 \quad \text{Simplify.}$$

Other information about a circle can be used to find the equation of the circle.

### Example 2 Use Characteristics of Circles

A circle with a diameter of 14 has its center in the third quadrant. The lines  $y = -1$  and  $x = 4$  are tangent to the circle. Write an equation of the circle.

Sketch a drawing of the two tangent lines.

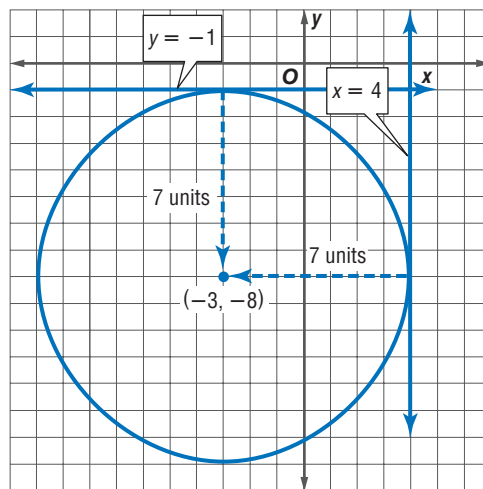
Since  $d = 14$ ,  $r = 7$ . The line  $x = 4$  is perpendicular to a radius. Since  $x = 4$  is a vertical line, the radius lies on a horizontal line. Count 7 units to the left from  $x = 4$ . Find the value of  $h$ .

$$h = 4 - 7 \text{ or } -3$$

Likewise, the radius perpendicular to the line  $y = -1$  lies on a vertical line. The value of  $k$  is 7 units down from  $-1$ .

$$k = -1 - 7 \text{ or } -8$$

The center is at  $(-3, -8)$ , and the radius is 7. An equation for the circle is  $(x + 3)^2 + (y + 8)^2 = 49$ .



**GRAPH CIRCLES** You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane.

### Study Tip

#### Graphing Calculator

You can also use the center and radius to graph a circle on a graphing calculator. Select a suitable window that contains the center of the circle. For a TI-83 Plus, press **ZOOM** 5. Then use **9: Circle** ( on the **Draw** menu. Put in the coordinates of the center and then the radius so that the screen shows "Circle  $(-2, 3, 4)$ ". Then press **ENTER**.

### Example 3 Graph a Circle

a. Graph  $(x + 2)^2 + (y - 3)^2 = 16$ .

Compare each expression in the equation to the standard form.

$$(x - h)^2 = (x + 2)^2 \qquad (y - k)^2 = (y - 3)^2$$

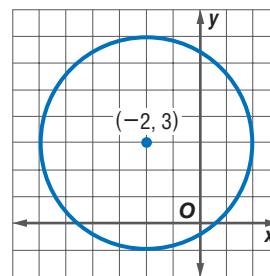
$$x - h = x + 2 \qquad y - k = y - 3$$

$$-h = 2 \qquad -k = -3$$

$$h = -2 \qquad k = 3$$

$$r^2 = 16, \text{ so } r = 4.$$

The center is at  $(-2, 3)$ , and the radius is 4. Graph the center. Use a compass set at a width of 4 grid squares to draw the circle.

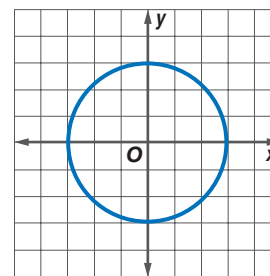


b. Graph  $x^2 + y^2 = 9$ .

Write the equation in standard form.

$$(x - 0)^2 + (y - 0)^2 = 3^2$$

The center is at  $(0, 0)$ , and the radius is 3. Draw a circle with radius 3, centered at the origin.



If you know three points on the circle, you can find the center and radius of the circle and write its equation.

### Example 4 A Circle Through Three Points

**CELL PHONES** Cell phones work by the transfer of phone signals from one tower to another via a satellite. Cell phone companies try to locate towers so that they service multiple communities. Suppose three large metropolitan areas are modeled by the points  $A(4, 4)$ ,  $B(0, -12)$ , and  $C(-4, 6)$ , and each unit equals 100 miles. Determine the location of a tower equidistant from all three cities, and write an equation for the circle.

**Explore** You are given three points that lie on a circle.

**Plan** Graph  $\triangle ABC$ . Construct the perpendicular bisectors of two sides to locate the center, which is the location of the tower. Find the length of a radius. Use the center and radius to write an equation.

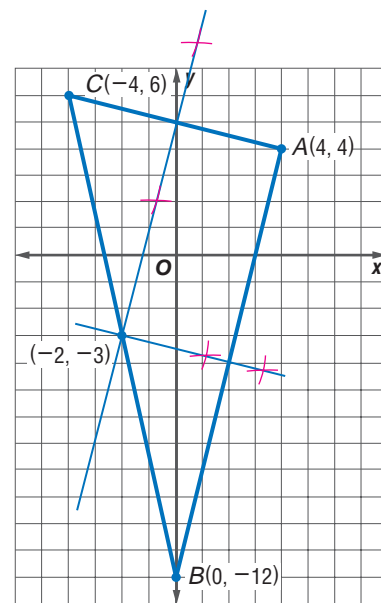
**Solve** Graph  $\triangle ABC$  and construct the perpendicular bisectors of two sides. The center appears to be at  $(-2, -3)$ . This is the location of the tower.

Find  $r$  by using the Distance Formula with the center and any of the three points.

$$\begin{aligned} r &= \sqrt{[-2 - 4]^2 + [-3 - 4]^2} \\ &= \sqrt{85} \end{aligned}$$

Write an equation.

$$\begin{aligned} [x - (-2)]^2 + [y - (-3)]^2 &= (\sqrt{85})^2 \\ (x + 2)^2 + (y + 3)^2 &= 85 \end{aligned}$$



**Examine** You can verify the location of the center by finding the equations of the two bisectors and solving a system of equations. You can verify the radius by finding the distance between the center and another of the three points on the circle.

#### Study Tip

**Look Back**  
To review **writing equations of lines**, see Lesson 5-4.

## Check for Understanding

- Concept Check**
- OPEN ENDED** Draw an obtuse triangle on a coordinate plane and construct the circle that circumscribes it.
  - Explain** how the definition of a circle leads to its equation.

**Guided Practice** Write an equation for each circle.

- center at  $(-3, 5)$ ,  $r = 10$
- center at origin,  $r = \sqrt{7}$
- diameter with endpoints at  $(2, 7)$  and  $(-6, 15)$

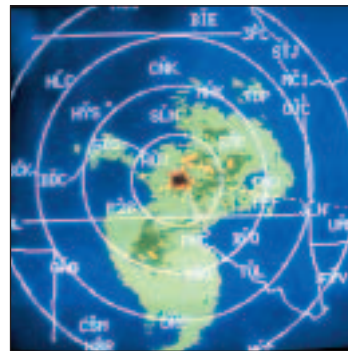
**Graph each equation.**

- $(x + 5)^2 + (y - 2)^2 = 9$
- $(x - 3)^2 + y^2 = 16$
- Write an equation of a circle that contains  $M(-2, -2)$ ,  $N(2, -2)$ , and  $Q(2, 2)$ . Then graph the circle.



## Application

9. **WEATHER** Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring?



## Practice and Apply

### Homework Help

For Exercises	See Examples
10–17	1
18–23	2
24–29	3
30–31	4

**Extra Practice**  
See page 776.

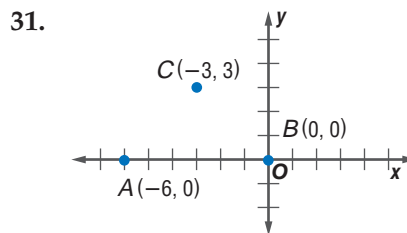
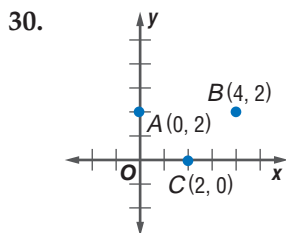
Write an equation for each circle.

- center at origin,  $r = 3$
- center at  $(1, -4)$ ,  $r = \sqrt{17}$
- center at  $(5, 10)$ ,  $r = 7$
- center at  $(-8, 8)$ ,  $d = 16$
- a circle with center at  $(-3, 6)$  and a radius with endpoint at  $(0, 6)$
- a circle with a diameter that has endpoints at  $(2, -2)$  and  $(-2, 2)$
- a circle with a diameter that has endpoints at  $(-7, -2)$  and  $(-15, 6)$
- a circle with center at  $(-2, 1)$  and a radius with endpoint at  $(1, 0)$
- a circle with  $d = 12$  and a center translated 18 units left and 7 units down from the origin
- a circle with its center in quadrant I, radius of 5 units, and tangents  $x = 2$  and  $y = 3$
- center at  $(-2, -8)$ ,  $r = 5$
- center at  $(0, 0)$ ,  $d = 12$
- center at  $(0, 5)$ ,  $d = 20$
- center at  $(-3, -10)$ ,  $d = 24$

Graph each equation.

- $x^2 + y^2 = 25$
- $x^2 + y^2 - 1 = 0$
- $(x - 2)^2 + (y - 1)^2 = 4$
- $x^2 + y^2 = 36$
- $x^2 + y^2 - 49 = 0$
- $(x + 1)^2 + (y + 2)^2 = 9$

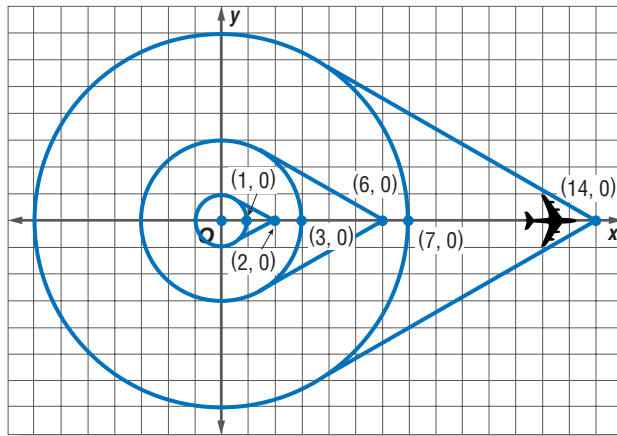
Write an equation of the circle containing each set of points. Copy and complete the graph of the circle.



- Find the radius of a circle with equation  $(x - 2)^2 + (y - 2)^2 = r^2$  that contains the point at  $(2, 5)$ .
- Find the radius of a circle with equation  $(x - 5)^2 + (y - 3)^2 = r^2$  that contains the point at  $(5, 1)$ .
- Refer to the Examine part of Example 4. Verify the coordinates of the center by solving a system of equations that represent the perpendicular bisectors.

**AERODYNAMICS** For Exercises 35–37, use the following information.

The graph shows cross sections of spherical sound waves produced by a supersonic airplane. When the radius of the wave is 1 unit, the plane is 2 units from the origin. A wave of radius 3 occurs when the plane is 6 units from the center.



35. Write the equation of the circle when the plane is 14 units from the center.
36. What type of circles are modeled by the cross sections?
37. What is the radius of the circle for a plane 26 units from the center?
38. The equation of a circle is  $(x - 6)^2 + (y + 2)^2 = 36$ . Determine whether the line  $y = 2x - 2$  is a secant, a tangent, or neither of the circle. Explain.
39. The equation of a circle is  $x^2 - 4x + y^2 + 8y = 16$ . Find the center and radius of the circle.
40. **WEATHER** The geographic center of Tennessee is near Murfreesboro. The closest Doppler weather radar is in Nashville. If Murfreesboro is designated as the origin, then Nashville has coordinates  $(-58, 55)$ , where each unit is one mile. If the radar has a radius of 80 miles, write an equation for the circle that represents the radar coverage from Nashville.
41. **RESEARCH** Use the Internet or other materials to find the closest Doppler radar to your home. Write an equation of the circle for the radar coverage if your home is the center.
42. **SPACE TRAVEL** Apollo 8 was the first manned spacecraft to orbit the moon at an average altitude of 185 kilometers above the moon's surface. Determine an equation to model a single circular orbit of the Apollo 8 command module if the radius of the moon is 1740 kilometers. Let the center of the moon be at the origin.
43. **CRITICAL THINKING** Determine the coordinates of any intersection point of the graphs of each pair of equations.
  - a.  $x^2 + y^2 = 9, y = x + 3$
  - b.  $x^2 + y^2 = 25, x^2 + y^2 = 9$
  - c.  $(x + 3)^2 + y^2 = 9, (x - 3)^2 + y^2 = 9$
44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**What kind of equations describe the ripples of a splash?**

Include the following in your answer:

- the general form of the equation of a circle, and
- the equations of five ripples if each ripple is 3 inches farther from the center.



**Space Travel**

The Apollo program was designed to successfully land a man on the moon. The first landing was July 20, 1969. There were a total of six landings on the moon during 1969–1972.

Source: www.infoplease.com



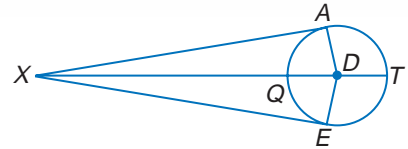


45. Which of the following is an equation of a circle with center at  $(-2, 7)$  and a diameter of 18?
- (A)  $x^2 + y^2 - 4x + 14y + 53 = 324$       (B)  $x^2 + y^2 + 4x - 14y + 53 = 81$   
 (C)  $x^2 + y^2 - 4x + 14y + 53 = 18$       (D)  $x^2 + y^2 + 4x - 14y + 53 = 3$
46. **ALGEBRA** Jordan opened a one-gallon container of milk and poured one pint of milk into his glass. What is the fractional part of one gallon left in the container?
- (A)  $\frac{1}{8}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{4}$       (D)  $\frac{7}{8}$

## Maintain Your Skills

**Mixed Review** Find each measure if  $EX = 24$  and  $DE = 7$ . (Lesson 10-7)

47.  $AX$       48.  $DX$   
 49.  $QX$       50.  $TX$



Find  $x$ . (Lesson 10-6)

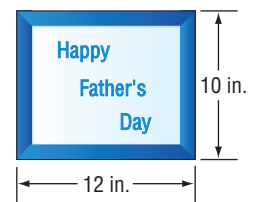
51.      52.      53.

For Exercises 54 and 55, use the following information.

Triangle  $ABC$  has vertices  $A(-3, 2)$ ,  $B(4, -1)$ , and  $C(0, -4)$ .

54. What are the coordinates of the image after moving  $\triangle ABC$  3 units left and 4 units up? (Lesson 9-2)
55. What are the coordinates of the image of  $\triangle ABC$  after a reflection in the  $y$ -axis? (Lesson 9-1)

56. **CRAFTS** For a Father's Day present, a kindergarten class is making foam plaques. The edge of each plaque is covered with felt ribbon all the way around with 1 inch overlap. There are 25 children in the class. How much ribbon does the teacher need to buy for all 25 children to complete this craft? (Lesson 1-6)



## WebQuest Internet Project

### "Geocaching" Sends Folks on a Scavenger Hunt

It's time to complete your project. Use the information and data you have gathered about designing a treasure hunt to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.



[www.geometryonline.com/webquest](http://www.geometryonline.com/webquest)

## Vocabulary and Concept Check

arc (p. 530)	circumference (p. 523)	major arc (p. 530)	radius (p. 522)
center (p. 522)	circumscribed (p. 537)	minor arc (p. 530)	secant (p. 561)
central angle (p. 529)	diameter (p. 522)	pi ( $\pi$ ) (p. 524)	semicircle (p. 530)
chord (p. 522)	inscribed (p. 537)	point of tangency (p. 552)	tangent (p. 552)
circle (p. 522)	intercepted (p. 544)		

A complete list of postulates and theorems can be found on pages R1–R8.

**Exercises** Choose the letter of the term that best matches each phrase.

- arcs of a circle that have exactly one point in common
- a line that intersects a circle in exactly one point
- an angle with a vertex that is on the circle and with sides containing chords of the circle
- a line that intersects a circle in exactly two points
- an angle with a vertex that is at the center of the circle
- arcs that have the same measure
- the distance around a circle
- circles that have the same radius
- a segment that has its endpoints on the circle
- circles that have different radii, but the same center

- adjacent arcs
- central angle
- chord
- circumference
- concentric circles
- congruent arcs
- congruent circles
- inscribed angle
- secant
- tangent

## Lesson-by-Lesson Review

## 10-1 Circles and Circumference

See pages  
522–528.

## Concept Summary

- The diameter of a circle is twice the radius.
- The circumference  $C$  of a circle with diameter  $d$  or a radius of  $r$  can be written in the form  $C = \pi d$  or  $C = 2\pi r$ .

## Example

Find  $r$  to the nearest hundredth if  $C = 76.2$  feet.

$$C = 2\pi r \quad \text{Circumference formula}$$

$$76.2 = 2\pi r \quad \text{Substitution}$$

$$\frac{76.2}{2\pi} = r \quad \text{Divide each side by } 2\pi.$$

$$12.13 \approx r \quad \text{Use a calculator.}$$

**Exercises** The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary. See Example 4 on page 524.

11.  $d = 15$  in.,  $r = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$

12.  $r = 6.4$  m,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$

13.  $C = 68$  yd,  $r = \underline{\quad ? \quad}$ ,  $d = \underline{\quad ? \quad}$

14.  $d = 52$  cm,  $r = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$

15.  $C = 138$  ft,  $r = \underline{\quad ? \quad}$ ,  $d = \underline{\quad ? \quad}$

16.  $r = 11$  mm,  $d = \underline{\quad ? \quad}$ ,  $C = \underline{\quad ? \quad}$



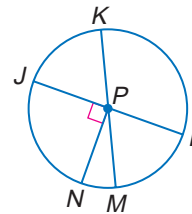
## 10-2 Angles and Arcs

See pages 529–535.

### Concept Summary

- The sum of the measures of the central angles of a circle with no interior points in common is 360.
- The measure of each arc is related to the measure of its central angle.
- The length of an arc is proportional to the length of the circumference.

**Examples** In  $\odot P$ ,  $m\angle MPL = 65$  and  $\overline{NP} \perp \overline{PL}$ .



1 Find  $m\widehat{NM}$ .

$\widehat{NM}$  is a minor arc, so  $m\widehat{NM} = m\angle NPM$ .  
 $\angle JPN$  is a right angle and  $m\angle MPL = 65$ ,  
 so  $m\angle NPM = 25$ .

$$m\widehat{NM} = 25$$

2 Find  $m\widehat{NJK}$ .

$\widehat{NJK}$  is composed of adjacent arcs,  $\widehat{NJ}$  and  $\widehat{JK}$ .  $\angle MPL \cong \angle JPK$ , so  $m\angle JPK = 65$ .

$$m\widehat{NJ} = m\angle NPJ \text{ or } 90 \quad \angle NPJ \text{ is a right angle}$$

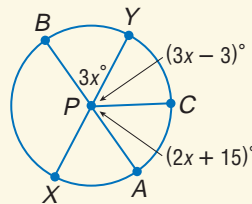
$$m\widehat{NJK} = m\widehat{NJ} + m\widehat{JK} \quad \text{Arc Addition Postulate}$$

$$m\widehat{NJK} = 90 + 65 \text{ or } 155 \quad \text{Substitution}$$

**Exercises** Find each measure.

See Example 1 on page 529.

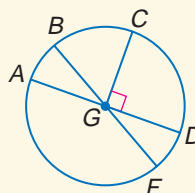
- $m\widehat{YC}$
- $m\widehat{BC}$
- $m\widehat{BX}$
- $m\widehat{BCA}$



In  $\odot G$ ,  $m\angle AGB = 30$  and  $\overline{CG} \perp \overline{GD}$ .

Find each measure. See Example 2 on page 531.

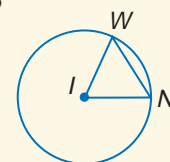
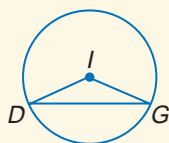
- |                      |                      |
|----------------------|----------------------|
| 21. $m\widehat{AB}$  | 22. $m\widehat{BC}$  |
| 23. $m\widehat{FD}$  | 24. $m\widehat{CDF}$ |
| 25. $m\widehat{BCD}$ | 26. $m\widehat{FAB}$ |



Find the length of the indicated arc in each  $\odot I$ . See Example 4 on page 532.

27.  $\widehat{DG}$  if  $m\angle DGI = 24$  and  $r = 6$

28.  $\widehat{WN}$  if  $\triangle IWN$  is equilateral and  $WN = 5$



## 10-3 Arcs and Chords

See pages  
536–543.

### Concept Summary

- The endpoints of a chord are also the endpoints of an arc.
- Diameters perpendicular to chords bisect chords and intercepted arcs.

### Examples

Circle  $L$  has a radius of 32 centimeters.  
 $\overline{LH} \perp \overline{GJ}$ , and  $GJ = 40$  centimeters. Find  $LK$ .

Draw radius  $\overline{LJ}$ .  $LJ = 32$  and  $\triangle LKJ$  is a right triangle.

$\overline{LH}$  bisects  $\overline{GJ}$ , since they are perpendicular.

$$KJ = \frac{1}{2}(GJ) \quad \text{Definition of segment bisector}$$

$$= \frac{1}{2}(40) \text{ or } 20 \quad GJ = 40, \text{ and simplify.}$$

Use the Pythagorean Theorem to find  $LK$ .

$$(LK)^2 + (KJ)^2 = (LJ)^2 \quad \text{Pythagorean Theorem}$$

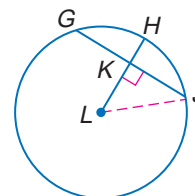
$$(LK)^2 + 20^2 = 32^2 \quad KJ = 20, LJ = 32$$

$$(LK)^2 + 400 = 1024 \quad \text{Simplify.}$$

$$(LK)^2 = 624 \quad \text{Subtract 400 from each side.}$$

$$LK = \sqrt{624} \quad \text{Take the square root of each side.}$$

$$LK \approx 24.98 \quad \text{Use a calculator.}$$



**Exercises** In  $\odot R$ ,  $SU = 20$ ,  $YW = 20$ , and  $m\widehat{YX} = 45$ .  
Find each measure. See Example 3 on page 538.

29.  $SV$

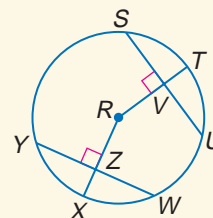
30.  $WZ$

31.  $UV$

32.  $m\widehat{YW}$

33.  $m\widehat{ST}$

34.  $m\widehat{SU}$



## 10-4 Inscribed Angles

See pages  
544–551.

### Concept Summary

- The measure of the inscribed angle is half the measure of its intercepted arc.
- The angles of inscribed polygons can be found by using arc measures.

### Example

**ALGEBRA** Triangles  $FGH$  and  $FJH$  are inscribed in  $\odot K$   
with  $\widehat{FG} \cong \widehat{FJ}$ . Find  $x$  if  $m\angle 1 = 6x - 5$ , and  $m\angle 2 = 7x + 4$ .

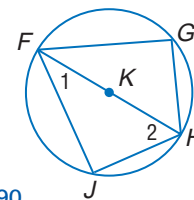
$FJH$  is a right angle because  $\widehat{FJH}$  is a semicircle.

$$m\angle 1 + m\angle 2 + m\angle FJH = 180 \quad \text{Angle Sum Theorem}$$

$$(6x - 5) + (7x + 4) + 90 = 180 \quad m\angle 1 = 6x - 5, m\angle 2 = 7x + 4, m\angle FJH = 90$$

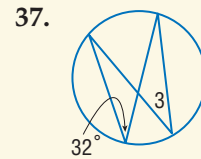
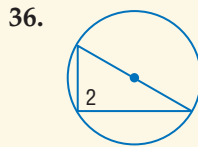
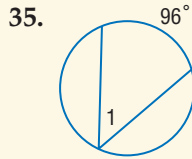
$$13x + 89 = 180 \quad \text{Simplify.}$$

$$x = 7 \quad \text{Solve for } x.$$



**Exercises** Find the measure of each numbered angle.

See Example 1 on page 545.



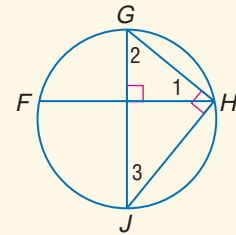
Find the measure of each numbered angle for each situation given.

See Example 4 on page 547.

38.  $m\widehat{GH} = 78$

39.  $m\angle 2 = 2x, m\angle 3 = x$

40.  $m\widehat{JH} = 114$



**10-5** Tangents

See pages 552–558.

**Concept Summary**

- A line that is tangent to a circle intersects the circle in exactly one point.
- A tangent is perpendicular to a radius of a circle.
- Two segments tangent to a circle from the same exterior point are congruent.

**Example**

**ALGEBRA** Given that the perimeter of  $\triangle ABC = 25$ , find  $x$ . Assume that segments that appear tangent to circles are tangent.

In the figure,  $\overline{AB}$  and  $\overline{AC}$  are drawn from the same exterior point and are tangent to  $\odot Q$ . So  $\overline{AB} \cong \overline{AC}$ .

The perimeter of the triangle,  $AB + BC + AC$ , is 25.

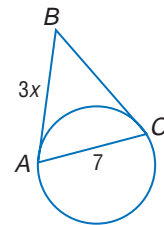
$AB + BC + AC = 25$  Definition of perimeter

$3x + 3x + 7 = 25$   $AB = BC = 3x, AC = 7$

$6x + 7 = 25$  Simplify.

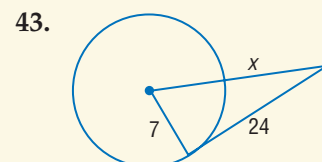
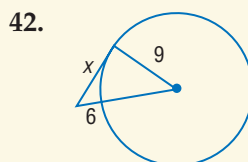
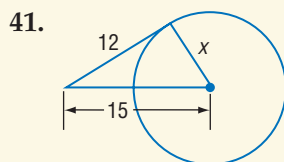
$6x = 18$  Subtract 7 from each side.

$x = 3$  Divide each side by 6.



**Exercises** Find  $x$ . Assume that segments that appear to be tangent are tangent.

See Example 3 on page 554.





## 10-6 Secants, Tangents, and Angle Measures

See pages 561–568.

### Concept Summary

- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

### Example

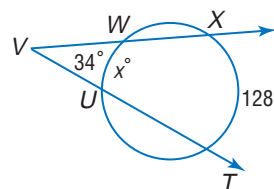
Find  $x$ .

$$m\angle V = \frac{1}{2}(m\widehat{XT} - m\widehat{WU})$$

$$34 = \frac{1}{2}(128 - x) \quad \text{Substitution}$$

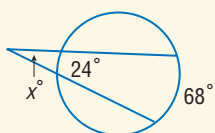
$$-30 = -\frac{1}{2}x \quad \text{Simplify.}$$

$$x = 60 \quad \text{Multiply each side by } -2.$$

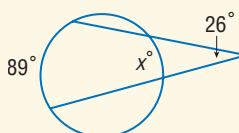


**Exercises** Find  $x$ . See Example 3 on page 563.

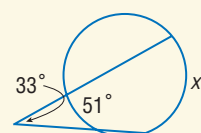
44.



45.



46.



## 10-7 Special Segments in a Circle

See pages 569–574.

### Concept Summary

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The secant segment product also applies to segments that intersect outside the circle, and to a secant segment and a tangent.

### Example

Find  $a$ , if  $FG = 18$ ,  $GH = 42$ , and  $FK = 15$ .

Let  $KJ = a$ .

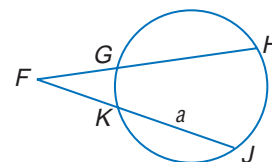
$$FK \cdot FJ = FG \cdot FH \quad \text{Secant Segment Products}$$

$$15(a + 15) = 18(18 + 42) \quad \text{Substitution}$$

$$15a + 225 = 1080 \quad \text{Distributive Property}$$

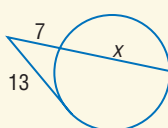
$$15a = 855 \quad \text{Subtract 225 from each side.}$$

$$a = 57 \quad \text{Divide each side by 15.}$$

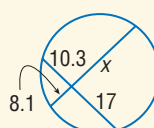


**Exercises** Find  $x$  to the nearest tenth. Assume that segments that appear to be tangent are tangent. See Examples 3 and 4 on pages 570 and 571.

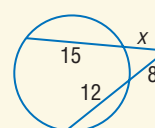
47.



48.



49.



## 10-8 Equations of Circles

See pages  
575–580.

### Concept Summary

- The coordinates of the center of a circle  $(h, k)$  and its radius  $r$  can be used to write an equation for the circle in the form  $(x - h)^2 + (y - k)^2 = r^2$ .
- A circle can be graphed on a coordinate plane by using the equation written in standard form.
- A circle can be graphed through any three noncollinear points on the coordinate plane.

### Examples

- 1 Write an equation of a circle with center  $(-1, 4)$  and radius 3.**

Since the center is at  $(-1, 4)$  and the radius is 3,  $h = -1$ ,  $k = 4$ , and  $r = 3$ .

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$[x - (-1)]^2 + (y - 4)^2 = 3^2 \quad h = -1, k = 4, \text{ and } r = 3$$

$$(x + 1)^2 + (y - 4)^2 = 9 \quad \text{Simplify.}$$

- 2 Graph  $(x - 2)^2 + (y + 3)^2 = 6.25$ .**

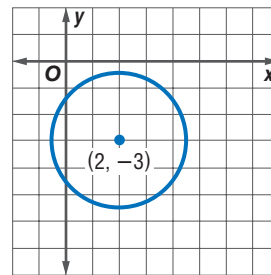
Identify the values of  $h$ ,  $k$ , and  $r$  by writing the equation in standard form.

$$(x - 2)^2 + (y + 3)^2 = 6.25$$

$$(x - 2)^2 + [y - (-3)]^2 = 2.5^2$$

$$h = 2, k = -3, \text{ and } r = 2.5$$

Graph the center  $(2, -3)$  and use a compass to construct a circle with radius 2.5 units.



**Exercises** Write an equation for each circle. See Examples 1 and 2 on pages 575 and 576.

50. center at  $(0, 0)$ ,  $r = \sqrt{5}$

51. center at  $(-4, 8)$ ,  $d = 6$

52. diameter with endpoints at  $(0, -4)$  and  $(8, -4)$

53. center at  $(-1, 4)$  and is tangent to  $x = 1$

**Graph each equation.** See Example 3 on page 576.

54.  $x^2 + y^2 = 2.25$

55.  $(x - 4)^2 + (y + 1)^2 = 9$

**For Exercises 56 and 57, use the following information.**

A circle graphed on a coordinate plane contains  $A(0, 6)$ ,  $B(6, 0)$ , and  $C(6, 6)$ .

See Example 4 on page 577.

56. Write an equation of the circle.

57. Graph the circle.

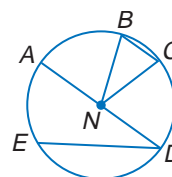
## Vocabulary and Concepts

1. Describe the differences among a tangent, a secant, and a chord of a circle.
2. Explain how to find the center of a circle given the coordinates of the endpoints of a diameter.

## Skills and Applications

3. Determine the radius of a circle with circumference  $25\pi$  units. Round to the nearest tenth.

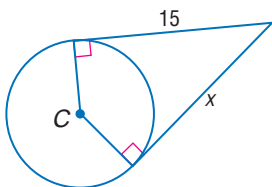
For Questions 4–11, refer to  $\odot N$ .



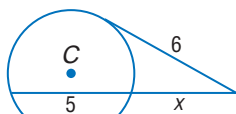
4. Name the radii of  $\odot N$ .
5. If  $AD = 24$ , find  $CN$ .
6. Is  $ED > AD$ ? Explain.
7. If  $AN$  is 5 meters long, find the exact circumference of  $\odot N$ .
8. If  $m\angle BNC = 20$ , find  $m\widehat{BC}$ .
9. If  $m\widehat{BC} = 30$  and  $\widehat{AB} \cong \widehat{CD}$ , find  $m\widehat{AB}$ .
10. If  $\widehat{BE} \cong \widehat{ED}$  and  $m\widehat{ED} = 120$ , find  $m\widehat{BE}$ .
11. If  $m\widehat{AE} = 75$ , find  $m\angle ADE$ .

Find  $x$ . Assume that segments that appear to be tangent are tangent.

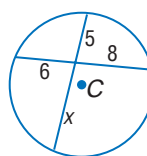
12.



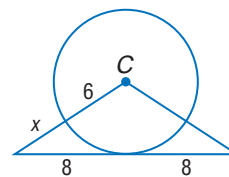
13.



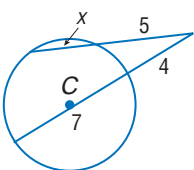
14.



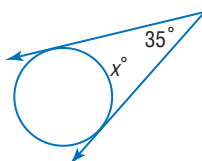
15.



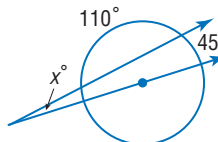
16.



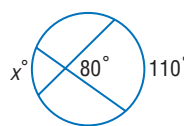
17.



18.



19.



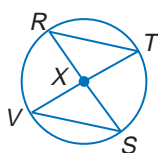
20. **AMUSEMENT RIDES** Suppose a Ferris wheel is 50 feet wide. Approximately how far does a rider travel in one rotation of the wheel?
21. Write an equation of a circle with center at  $(-2, 5)$  and a diameter of 50.
22. Graph  $(x - 1)^2 + (y + 2)^2 = 4$ .

23. **PROOF** Write a two-column proof.

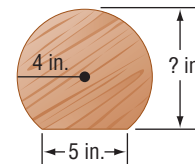
**Given:**  $\odot X$  with diameters

$\overline{RS}$  and  $\overline{TV}$

**Prove:**  $\widehat{RT} \cong \widehat{VS}$



24. **CRAFTS** Takita is making bookends out of circular wood pieces as shown at the right. What is the height of the cut piece of wood?



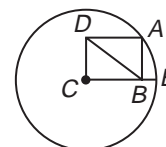
25. **STANDARDIZED TEST PRACTICE** Circle  $C$  has radius  $r$  and  $ABCD$  is a rectangle. Find  $DB$ .

(A)  $r$

(B)  $r\frac{\sqrt{2}}{2}$

(C)  $r\sqrt{3}$

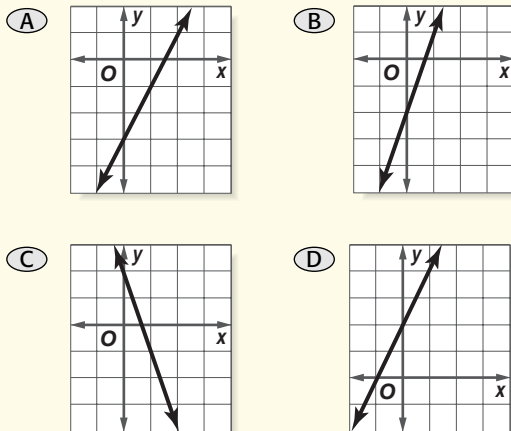
(D)  $r\frac{\sqrt{3}}{2}$



**Part 1 Multiple Choice**

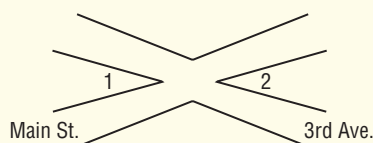
Record your answer on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following shows the graph of  $3y = 6x - 9$ ? (Prerequisite Skill)



2. In Hyde Park, Main Street and Third Avenue do not meet at right angles. Use the figure below to determine the measure of  $\angle 1$  if  $m\angle 1 = 6x - 5$  and  $m\angle 2 = 3x + 13$ . (Lesson 1-5)

- (A) 6
- (B) 18
- (C) 31
- (D) 36



3. Part of a proof is shown below. What is the reason to justify Step b? (Lesson 2-5)

**Given:**  $4x + \frac{4}{3} = 12$       **Prove:**  $x = \frac{8}{3}$

Statements	Reasons
a. $4x + \frac{4}{3} = 12$	a. Given
b. $3(4x + \frac{4}{3}) = 3(12)$	b. ?

- (A) Multiplication Property
- (B) Distributive Property
- (C) Cross products
- (D) none of the above

4. If an equilateral triangle has a perimeter of  $(2x + 9)$  miles and one side of the triangle measures  $(x + 2)$  miles, how long (in miles) is the side of the triangle? (Lesson 4-1)

- (A) 3
- (B) 5
- (C) 9
- (D) 15

5. A pep team is holding up cards to spell out the school name. What symmetry does the card shown below have? (Lesson 9-1)

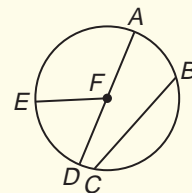
- (A) only line symmetry
- (B) only point symmetry
- (C) both line and point symmetry
- (D) neither line nor point symmetry



Use the figure below for Questions 6 and 7.

6. In circle  $F$ , which are chords? (Lesson 10-1)

- (A)  $\overline{AD}$  and  $\overline{EF}$
- (B)  $\overline{AF}$  and  $\overline{BC}$
- (C)  $\overline{EF}$ ,  $\overline{DF}$ , and  $\overline{AF}$
- (D)  $\overline{AD}$  and  $\overline{BC}$



7. In circle  $F$ , what is the measure of  $\widehat{EA}$  if  $m\angle DFE$  is  $36^\circ$ ? (Lesson 10-2)

- (A) 54
- (B) 104
- (C) 144
- (D) 324

8. Which statement is false? (Lesson 10-3)

- (A) Two chords that are equidistant from the center of a circle are congruent.
- (B) A diameter of a circle that is perpendicular to a chord bisects the chord and its arc.
- (C) The measure of a major arc is the measure of its central angle.
- (D) Minor arcs in the same circle are congruent if their corresponding chords are congruent.

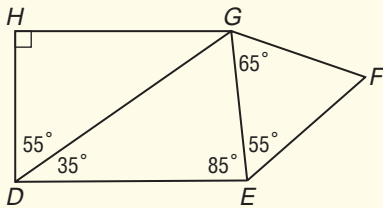
9. Which of the segments described could be a secant of a circle? (Lesson 10-6)

- (A) intersects exactly one point on a circle
- (B) has its endpoints on a circle
- (C) one endpoint at the center of the circle
- (D) intersects exactly two points on a circle

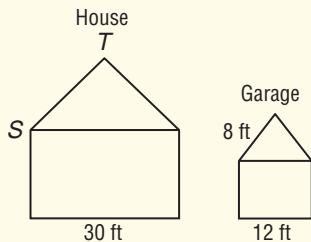
## Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. What is the shortest side of quadrilateral  $DEFG$ ? (Lesson 5-3)



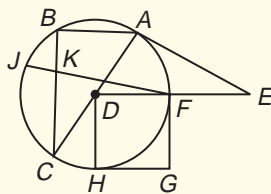
11. An architect designed a house and a garage that are proportional in shape. How many feet long is  $\overline{ST}$ ? (Lesson 6-2)



12. Two triangles are drawn on a coordinate grid. One has vertices at  $(0, 1)$ ,  $(0, 7)$ , and  $(6, 4)$ . The other has vertices at  $(7, 7)$ ,  $(10, 7)$ , and  $(8.5, 10)$ . What scale factor can be used to compare the smaller triangle to the larger? (Lesson 9-5)

Use the figure below for Questions 13–15.

13. Point  $D$  is the center of the circle. What is  $m\angle ABC$ ? (Lesson 10-4)



14.  $\overline{AE}$  is a tangent. If  $AD = 12$  and  $FE = 18$ , how long is  $\overline{AE}$  to the nearest tenth unit? (Lesson 10-5)
15. Chords  $\overline{JF}$  and  $\overline{BC}$  intersect at  $K$ . If  $BK = 8$ ,  $KC = 12$ , and  $KF = 16$ , find  $JK$ . (Lesson 10-7)



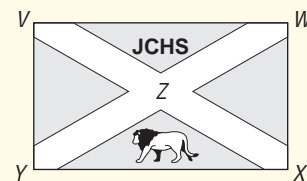
## Test-Taking Tip

**Question 4** If a question does not provide you with a figure that represents the problem, draw one yourself. By drawing and labeling an equilateral triangle with all the information you know, the problem becomes more understandable.

## Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

16. The Johnson County High School flag is shown below. Points have been added for reference.



- a. Which diagonal segments would have to be congruent for  $VWXY$  to be a rectangle? (Lesson 8-3)
- b. Suppose the length of rectangle  $VWXY$  is 2 more than 3 times the width and the perimeter is 164 inches. What are the dimensions of the flag? (Lesson 1-6)
17. A segment with endpoints  $A(1, -2)$  and  $B(1, 6)$  is the diameter of a circle.
- a. Graph the points and draw the circle. (Lesson 10-1)
- b. What is the center of the circle? (Lesson 10-1)
- c. What is the length of the radius? (Lesson 10-8)
- d. What is the circumference of the circle? (Lesson 10-8)
- e. What is the equation of the circle? (Lesson 10-8)